VCU

MATH 200

CALCULUS I

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TEST 3

April 20, 2016

Name: _____

Score: _____

Directions. Answer the questions in the provided space. Unless noted otherwise, you must show and explain your work to receive full credit. Put your final answer in a box when appropriate.

This is a closed-book, closed-notes test. Calculators, computers, etc., are not used. Please put all phones away.

1. (25 points) Find the indefinite integrals.

(a) $\int \left(x^3 + 3x + 5 \right) dx =$

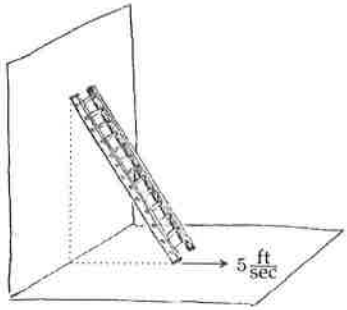
(b) $\int \frac{1}{\sqrt{x}} dx =$

(c) $\int \frac{e^x + 1}{e^x} dx =$

(d) $\int \frac{1}{1 + x^2} dx =$

(e) $\int \left(\sec^2(x) + 3\sin(x) \right) dx =$

2. (15 pts.) A 13-foot ladder leans against a wall, as shown. Its base slides away from the wall at a rate of 5 feet per second. How fast is the top of the ladder sliding down the wall when its base is 12 feet from the wall?



3. (15 pts.) Consider the function $f(x) = 5x^4 + 20x^3 + 10$.

(a) Find the critical points of $f(x)$.

(b) Find the intervals on which $f(x)$ increases/decreases

(c) State the locations of the local minima of $f(x)$ (if any).

(d) State the locations of the local maxima of $f(x)$ (if any).

(e) State the interval(s) on which $f(x)$ is concave down.

4. (15 pts.) Use L'Hôpital's rule to find the limits.

(a) $\lim_{x \to 0} \dfrac{e^{3x} - 1}{x^2 + 3x} =$

(b) $\lim_{x \to \frac{\pi}{2}} \left(\dfrac{\pi}{2} - x\right) \sec(x) =$

5 (15 pts.) USPS rules say the length plus girth of a package cannot exceed 108 inches. (Girth $= 2 \cdot \text{width} + 2 \cdot \text{height}$, as illustrated.) You must mail a package whose width and height are equal, and with the greatest possible volume. Find the dimensions of the package.

6. (15 pts.) A ball, tossed straight up, has a constant acceleration of -32 feet per second per second. At time $t = 0$ its velocity is $v(0) = 20$ feet per second, and its position is $s(0) = 5$ feet. Find the position function $s(t)$.