VCU

MATH 200

CALCULUS I

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TEST 2

☞

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Name: _____

Score: _____

Directions. Answer the questions in the provided space. Unless noted otherwise, you must show and explain your work to receive full credit. Put your final answer in a ☐ box ☐ when appropriate.

This is a closed-book, closed-notes test. Calculators, computers, etc., are not used. Please put all phones away.

1. (20 points) Warmup: short answer.

(a) If $f(x) = \tan^{-1}(x) + \dfrac{1}{x}$, then $f'(x) =$

(b) If $f(x) = \tan(x) + e^x$, then $f'(x) =$

(c) If $f(x) = \sqrt[5]{x^6}$, then $f'(x) =$

(d) If $f(x) = \frac{1}{2}\sin(x) + e$, then $f'(x) =$

(e) If $f(x) = e^{\pi x}$, then $f'(x) =$

(f) If $f(x) = e^{\pi x}$, then $f'(0) =$

(g) $\displaystyle\lim_{h \to 0} \frac{e^{x+h} - e^x}{h} =$

(h) $\dfrac{d}{dx}\left[\sin^{-1}(\pi x) \right] =$

(i) $\dfrac{d}{dx}\left[\ln\left(\sin(x) \right) \right] =$

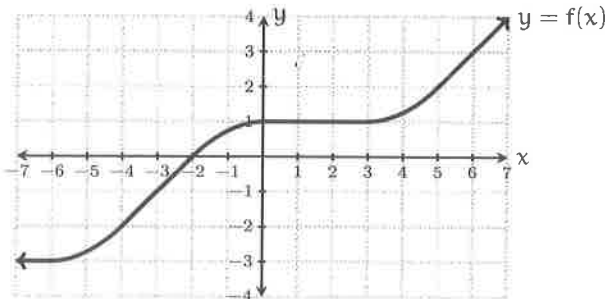(j) $\dfrac{d}{dx}\left[\dfrac{1}{x^2 + 3x} \right] =$

2. (5 points) Find the equation of the tangent line to the graph of $y = \sin(x)$ at the point where $x = 2\pi$.

3. (5 points) Information about a function $f(x)$ and its derivative is given in the table below.

x	0	1	2	3	4	5
$f(x)$	0	-3	-2	3	10	25
$f'(x)$	-1	-7	-5	5	20	30

Suppose $h(x) = f(x^2)$. Find $h'(2)$. Show your work.

4. (5 points) A function $f(x)$ is graphed below. Using the same coordinate axis, sketch the graph of the derivative $f'(x)$.

5. (20 points) Find the following derivatives.

(a) $\dfrac{d}{dx} \left[\sec \left(e^{3x+1} \right) \right] =$

(b) $\dfrac{d}{dx} \left[\ln \left(x^{10} - 4x^2 + 1 \right) \right] =$

(c) $\dfrac{d}{dx} \left[\tan(x^5) + \tan^5(x) \right] =$

(d) $\dfrac{d}{dx} \left[\dfrac{x^3 \ln(x)}{x^3 + 1} \right] =$

6 (10 points) Find the derivative of $y = x^{\ln(x)}$.

7 (10 points) Consider $f(x) = x^3 - 27x + 5$. Find all x for which the tangent to $y = f(x)$ at the point $(x, f(x))$ is horizontal.

8. (15 points) An object moves on a straight line in such a way that its distance from its starting point at time t seconds is $s(t) = 4\sqrt{t^5}$ feet. What is its velocity is when its acceleration is 30 feet per second per second?

9. (10 points) This question concerns the equation $x\sqrt[3]{y^2} + y = 12$.

(a) Use implicit differentiation to find $\dfrac{dy}{dx}$.

(b) Use your answer from part (a) to find the slope of the tangent line to the graph of $x\sqrt[3]{y^2} + y = 12$ at the point $(1, 8)$.