VCU

MATH 200

Calculus I

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Test 1

☞

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Name: Richard

Score: 100

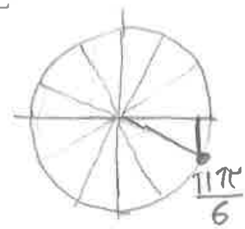
Directions. Answer the questions in the provided space. Unless noted otherwise, you must show and explain your work to receive full credit. Put your final answer in a $\boxed{\text{box}}$ when appropriate.

This is a closed-book, closed-notes test. Calculators, computers, etc., are not used. Please put all phones away.

1. (20 points) Warmup: short answer.

(a) $(-27)^{2/3} = \sqrt[3]{-27}^{\,2} = (-3)^2 = \boxed{9}$

(b) $\sin\left(\frac{11\pi}{6}\right) = \boxed{-\frac{1}{2}}$



(c) $\log_5(5) = 5^{\square}(5) = \boxed{1}$

(d) $\ln\left(\frac{1}{\sqrt{e}}\right) = \ln\left(\frac{1}{e^{1/2}}\right) = \ln\left(e^{-\frac{1}{2}}\right) = e^{\square}\left(e^{-\frac{1}{2}}\right) = \boxed{\frac{-1}{2}}$

(e) $\ln(\sin(\pi/2)) = \ln(1) = e^{\square}(1) = \boxed{0}$

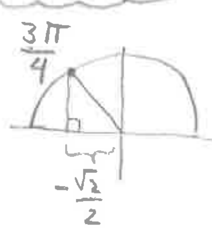
(f) $\lim_{x\to 0} \frac{\sin(x)}{x} = \boxed{1}$

(g) $\lim_{x\to \pi} \frac{\sin(x)}{x} = \frac{\sin(\pi)}{\pi} = \frac{0}{\pi} = \boxed{0}$

(h) $\lim_{x\to \infty} \frac{\sin(x)}{x} = \boxed{0}$

Reason numerator sin(x) always between −1 and 1 but denominator gets very large

(i) $\cos^{-1}\left(-\frac{\sqrt{2}}{2}\right) = \boxed{\frac{3\pi}{4}}$

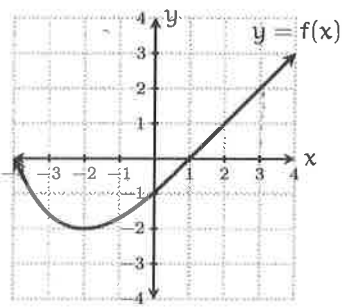


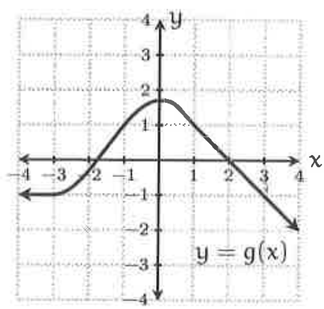
(j) $\lim_{x\to\pi} \cos(x) = \cos(\pi) = \boxed{-1}$

2. (10 points) For the functions $f(x)$ and $g(x)$ graphed below, find

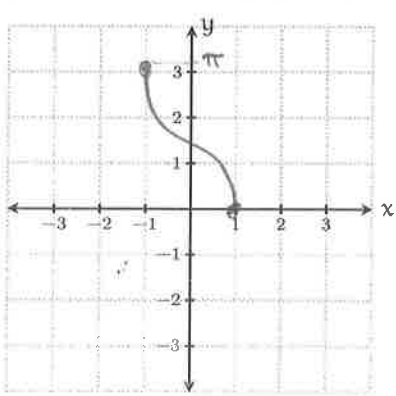
(a) $\lim_{x \to 3} \sqrt{5f(x) + g(x)} = \sqrt{5 \cdot 2 + (-1)} = \sqrt{9} = \boxed{3}$

(b) $\lim_{x \to 1} f(g(x)) = f\left(\lim_{x \to 1} g(x)\right) = f(1) = \boxed{0}$





3. (5 points) Sketch the graph of $y = \cos^{-1}(x)$.



(This is the graph of $y = \cos(x)$ for $0 \le x \le \pi$, reflected across the line $y = x$)

4. (20 points) Find the following limits.

(a) $\lim\limits_{x\to 2} \dfrac{x^3-4x}{x^2+x-6} = \lim\limits_{x\to 2} \dfrac{x(x^2-4)}{(x+3)(x-2)}$

$= \lim\limits_{x\to 2} \dfrac{x(x+2)\cancel{(x-2)}}{(x+3)\cancel{(x-2)}} = \lim\limits_{x\to 2} \dfrac{x(x+2)}{x+3}$

$= \dfrac{2(2+2)}{2+3} = \boxed{\dfrac{8}{5}}$

(b) $\lim\limits_{x\to 9} \dfrac{\sqrt{x}-3}{x-9} \cdot \dfrac{\sqrt{x}+3}{\sqrt{x}+3} = \lim\limits_{x\to 9} \dfrac{\cancel{x-9}}{\cancel{(x-9)}(\sqrt{x}+3)}$

$= \lim\limits_{x\to 9} \dfrac{1}{\sqrt{x}+3} = \dfrac{1}{\sqrt{9}+3} = \dfrac{1}{3+3} = \boxed{\dfrac{1}{6}}$

(c) $\lim\limits_{h\to 0} \dfrac{\dfrac{1}{(4+h)^2} - \dfrac{1}{16}}{h} \cdot \dfrac{16(4+h)^2}{16(4+h)^2} = \lim\limits_{h\to 0} \dfrac{16-(4+h)^2}{h\,16(4+h)^2}$

$= \lim\limits_{h\to 0} \dfrac{16-(16+8h+h^2)}{h\,16\,(4+h)^2}$

$= \lim\limits_{h\to 0} \dfrac{16-16-8h-h^2}{h\,16\,(4+h)^2} = \lim\limits_{h\to 0} \dfrac{-\cancel{h}(8+h)}{\cancel{h}\,16\,(4+h)^2}$

$= \lim\limits_{h\to 0} \dfrac{-(8+h)}{16(4+h)^2} = \dfrac{-(8+0)}{16(4+0)^2} = \dfrac{-8}{16\cdot 16} = \boxed{\dfrac{-1}{32}}$

(d) $\lim\limits_{x\to\infty} \tan^{-1}\left(\dfrac{x^2+3x-1}{x^2-2}\right) = \tan^{-1}\left(\lim\limits_{x\to\infty} \dfrac{x^2+3x-1}{x^2-2}\right)$

$= \tan^{-1}\left(\lim\limits_{x\to\infty} \dfrac{x^2+3x+1}{x^2-2} \cdot \dfrac{1/x^2}{1/x^2}\right)$

$= \tan^{-1}\left(\lim\limits_{x\to\infty} \dfrac{1+3/x+3/x^2}{1-2/x^2}\right) = \tan^{-1}\left(\dfrac{1+0+0}{1-0}\right)$

$= \tan^{-1}(1) = \boxed{\pi/4}$

5. (15 points) Sketch the graph of a function that meets all of the following criteria.

(a) The domain of $f(x)$ is all real numbers except $x = 0$

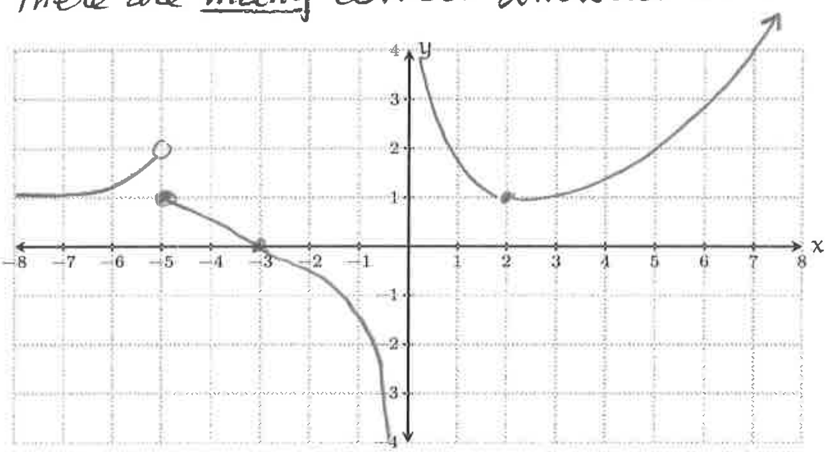
(b) $f(x)$ is continuous at all real numbers except $x = -5$ and $x = 0$

(c) $f(-3) = 0$ and $f(2) = 1$

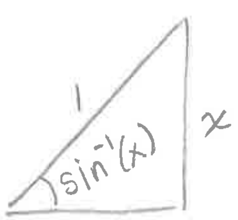
(d) $\lim\limits_{x\to\infty} f(x) = \infty$ and $\lim\limits_{x\to-\infty} f(x) = 1$

(e) $\lim\limits_{x\to0^+} f(x) = \infty$, and $\lim\limits_{x\to0^-} f(x) = -\infty$

There are <u>many</u> correct answers. Here's one:



6. (5 points) Simplify: $\tan(\sin^{-1}(x)) = \dfrac{\text{OPP}}{\text{ADJ}} = \boxed{\dfrac{x}{\sqrt{1-x^2}}}$



$\sqrt{1-x^2}$ by pythogorean theorem

7. (5 points) Find the inverse of the function $f(x) = \frac{\ln(3x+1)}{3}$.

$$y = \frac{\ln(3x+1)}{3}$$

$$x = \frac{\ln(3y+1)}{3}$$

$$3x = \ln(3y+1)$$

$$e^{3x} = e^{\ln(3y+1)}$$

$$e^{3x} = 3y+1$$

$$3y = e^{3x} - 1$$

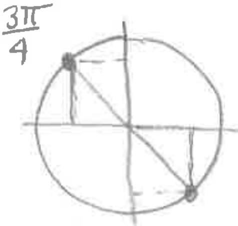
$$y = \frac{e^{3x}-1}{3}$$

$$\boxed{f^{-1}(x) = \frac{e^{3x}-1}{3}}$$

8. (10 points) Find all solutions of the equation $\sin(x) + \cos(x) = 0$.

These are the values of x for which $\sin(x) = -\cos(x)$. From the unit circle, we see that they are

$\frac{3\pi}{4}$



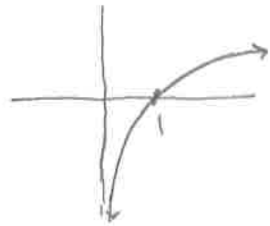
$$\boxed{x = \frac{3\pi}{4} + k\pi, \quad k = 0, \pm 1, \pm 2, \ldots}$$

9. (10 points) State the horizontal and vertical asymptotes of the following functions. You do not need to show any work. If there is no asymptote, write "none."

(a) $y = \ln(x)$

Horizontal: NONE

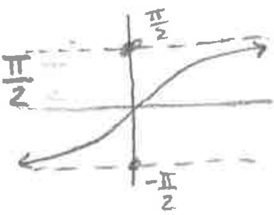
Vertical: Line $x = 0$



(b) $y = \tan^{-1}(x)$

Horizontal: Lines $y = \frac{\pi}{2}$ & $y = -\frac{\pi}{2}$

Vertical: NONE



(c) $y = \dfrac{x+1}{3x-2}$

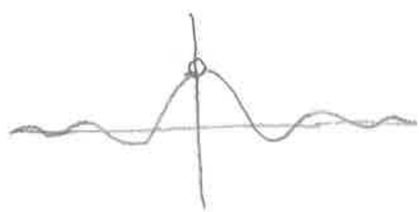
Horizontal: Line $y = \frac{1}{3}$

Vertical: Line $x = \frac{2}{3}$

(c) $y = \dfrac{\sin(x)}{x}$

Horizontal: Line $y = 0$

Vertical: NONE



(d) $y = e^x$

Horizontal: Line $y = 0$

Vertical: NONE