

VCU
MATH 200
CALCULUS I

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TEST 3



April 17, 2015

Name: _____

Score: _____

Directions. Answer the questions in the space provided. To get full credit, please show and explain your work as appropriate. Put your final answer in a when appropriate.

This is a closed-book, closed-notes test. Calculators, computers, etc., are not used.

1. (25 points) Find the indefinite integrals.

$$(a) \int \left(x^4 + \frac{1}{x} + \sqrt{2} \right) dx =$$

$$(b) \int \frac{1}{x^2} dx =$$

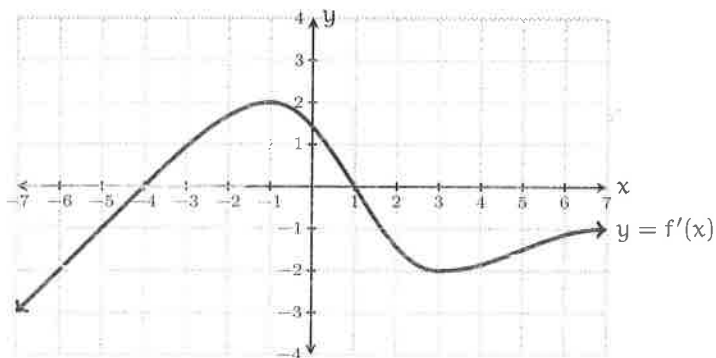
$$(c) \int e^{2x} dx =$$

$$(d) \int 3 \sec(x) \tan(x) dx =$$

(e) If $f(x)$ and $g(x)$ are differentiable functions, then

$$\int \frac{f'(x)g(x) - f(x)g'(x)}{(g(x))^2} dx =$$

2. (15 pts.) The graph of the derivative $f'(x)$ of a function $f(x)$ is sketched below. Answer the following questions about the function $f(x)$.



- (a) List the critical points of $f(x)$.
- (b) State the interval(s) on which $f(x)$ increases.
- (c) State the interval(s) on which $f(x)$ decreases.
- (d) State the locations of the relative extrema of $f(x)$.
- (e) State the interval(s) on which $f(x)$ is concave down.

3. (15 pts.) Consider the function $f(x) = x^2e^{-x}$.

(a) Find the critical points of $f(x)$.

(b) Find the intervals on which $f(x)$ increases.

(c) Find the intervals on which $f(x)$ decreases.

(d) State the locations of the local maxima of $f(x)$.

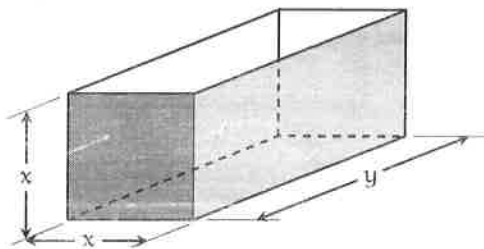
(e) State the locations of the local minima of $f(x)$.

4. (20 pts.) Use L'Hôpital's rule to find the limits.

(a) $\lim_{x \rightarrow \pi} \frac{\sin(x)}{x^2 - \pi^2} =$

(b) $\lim_{x \rightarrow \infty} \left(1 + \frac{1}{x}\right)^x =$

- 5 (15 pts.) A metal box with two square ends and an open top is to contain a volume of 36 cubic inches. What dimensions x and y will minimize the total area of the metal surface?



6. (10 pts.) Suppose $f(x)$ is a function for which

$$f'(x) = \frac{3}{\sqrt[3]{x^2}} \text{ and } f(-1) = -5. \text{ Find } f(x).$$