VCU

MATH 200

Calculus I

R. Hammack

Test 1

✍

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Name: Richard

Score: 100

Directions. Answer the questions in the space provided. Unless noted otherwise, you must show and explain your work to receive full credit. Put your final answer in a $\boxed{\text{box}}$ when appropriate.

This is a closed-book, closed-notes test. Calculators, computers, etc., are not used.

1. (25 points) Warmup: quick answer.

(a) $(-8)^{\frac{1}{3}} = \sqrt[3]{-8} = \boxed{-2}$

(b) State the domain of $f(x) = \dfrac{\sqrt{x+1}}{x^2-5}$.

Must have $x+1 \geq 0 \rightarrow x \geq -1$

Must have $x^2-5 \neq 0 \rightarrow x \neq \pm\sqrt{5}$

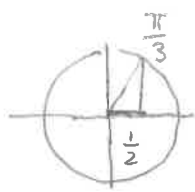
$\boxed{\text{Domain } [-1, \sqrt{5}) \cup (\sqrt{5}, \infty)}$

(c) If $f(x) = x + \dfrac{1}{x}$ and $g(x) = \sqrt{x}$, then:

$f \circ g(x) = f(g(x)) = \boxed{\sqrt{x} + \dfrac{1}{\sqrt{x}}}$

$g \circ f(x) = \boxed{\sqrt{x + \dfrac{1}{x}}}$

(d) $\cos\left(\frac{\pi}{3}\right) = \boxed{\dfrac{1}{2}}$



(e) $\lim\limits_{x\to\frac{\pi}{3}} (7+2\cos(x))^{\frac{2}{3}} = \lim\limits_{x\to\frac{\pi}{3}} \sqrt[3]{7+2\cos(x)}^{\,2}$

$= \sqrt[3]{\lim\limits_{x\to\frac{\pi}{3}}(7+2\cos(x))}^{\,2} = \sqrt[3]{7+2\cos\left(\frac{\pi}{3}\right)}^{\,2}$

$= \sqrt[3]{7+2\cdot\frac{1}{2}}^{\,2} = \sqrt[3]{8}^{\,2} = 2^2 = \boxed{4}$

2. (10 points) Consider the equation $2\cos(x)\sin(x) = \sin(x)$.
Find all solutions x of this equation for which $0 \leqslant x \leqslant 2\pi$.

$$2\cos(x)\sin(x) - \sin(x) = 0$$

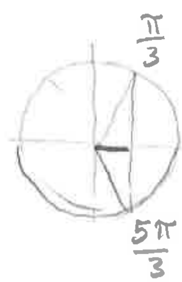
$$\sin(x)\left(2\cos(x) - 1\right) = 0$$

$\sin(x) = 0$

$2\cos(x) - 1 = 0$

$2\cos(x) = 1$

$\cos(x) = \frac{1}{2}$



Answer Solutions are:

$$\boxed{x = 0, \quad x = \frac{\pi}{3}, \quad x = \pi, \quad x = \frac{5\pi}{3}, \quad x = 2\pi}$$

3. (15 points) Evaluate the following limits.

(a) $\lim_{x\to 3} \frac{x^2-8x+15}{x^2-2x-3} = \lim_{x\to 3} \frac{\cancel{(x-3)}(x-5)}{\cancel{(x-3)}(x+1)}$

$= \lim_{x\to 3} \frac{x-5}{x+1} = \frac{3-5}{3+1} = \frac{-2}{4} = \boxed{-\frac{1}{2}}$

$\frac{0}{0}$, so try to cancel

(b) $\lim_{x\to 0} \frac{\sin(\sqrt{9x})}{\sqrt{x}} = \lim_{x\to 0} \frac{3\sin(\sqrt{9x})}{3\sqrt{x}}$

Idea: Use $\lim_{x\to 0} \frac{\sin x}{x} = 1$

$= 3 \lim_{x\to 0} \frac{\sin(\sqrt{9x})}{\sqrt{9x}}$

$= 3 \cdot 1 = \boxed{3}$

(c) $\lim_{x\to 2} \frac{\sqrt{x^2+12}-4}{x-2} = \lim_{x\to 2} \frac{\sqrt{x^2+12}-4}{x-2} \cdot \frac{\sqrt{x^2+12}+4}{\sqrt{x^2+12}+4}$

$\frac{0}{0}$, so try to cancel

$= \lim_{x\to 2} \frac{x^2+12-16}{(x-2)(\sqrt{x^2+12}+4)}$

$= \lim_{x\to 2} \frac{x^2-4}{(x-2)(\sqrt{x^2+12}+4)}$

$= \lim_{x\to 2} \frac{\cancel{(x-2)}(x+2)}{\cancel{(x-2)}(\sqrt{x^2+12}+4)} = \frac{2+2}{\sqrt{2^2+12}+4}$

$= \frac{4}{\sqrt{16}+4} = \frac{4}{4+4} = \boxed{\frac{1}{2}}$

4. (15 points) Sketch the graph of any function that meets all of the following criteria.

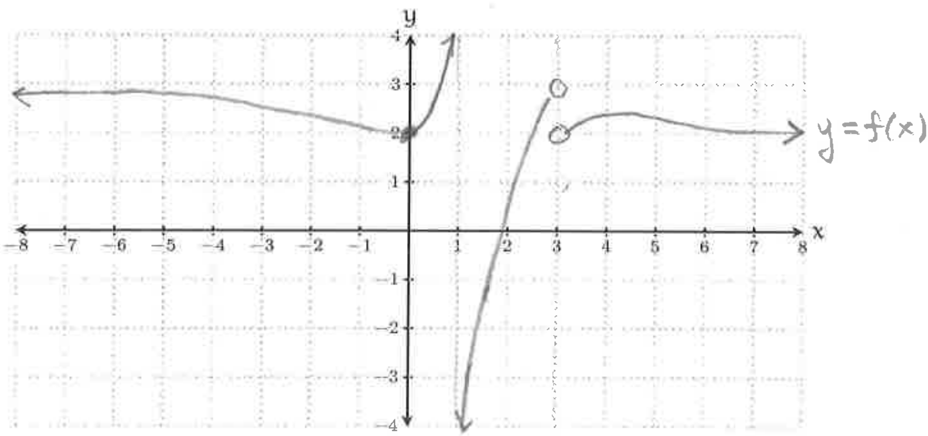
(a) $f(0) = 2$

(b) $f(x)$ is continuous at all real numbers except $x = 1$ and $x = 3$

(c) $\lim_{x \to 1^-} f(x) = \infty$ and $\lim_{x \to 1^+} f(x) = -\infty$

(d) $\lim_{x \to 3^-} f(x) = 3$ and $\lim_{x \to 3^+} f(x) = 2$

(e) $\lim_{x \to -\infty} f(x) = 3$ and $\lim_{x \to \infty} f(x) = 2$



5. (20 points) Evaluate the following limits.

(a) $\lim\limits_{x\to 0}\left(\frac{\sin(x)}{x}+\frac{1}{x-1}\right)=\lim\limits_{x\to 0}\frac{\sin(x)}{x}+\lim\limits_{x\to 0}\frac{1}{x-1}$

$=1+\frac{1}{0-1}=1-1=\boxed{0}$

(b) $\lim\limits_{x\to 1^+}\left(\frac{\sin(x)}{x}+\frac{1}{x-1}\right)=\lim\limits_{x\to 1^+}\frac{\sin(x)}{x}+\lim\limits_{x\to 1^+}\frac{1}{x-1}$

$=\frac{\sin(1)}{1}+\lim\limits_{x\to 1^+}\frac{1}{x-1}$

approaches 0, positive

$=\boxed{\infty}$

(c) $\lim\limits_{x\to\infty}\left(\frac{\sin(x)}{x}+\frac{1}{x-1}\right)=\lim\limits_{x\to\infty}\frac{\sin(x)}{x}+\lim\limits_{x\to\infty}\frac{1}{x-1}$

$=0+0=\boxed{0}$

(d) $\lim\limits_{x\to\pi}\left(\frac{\sin(x)}{x}+\frac{1}{x-1}\right)=\lim\limits_{x\to\pi}\frac{\sin(x)}{x}+\lim\limits_{x\to\pi}\frac{1}{x-1}$

$=\frac{\sin(\pi)}{\pi}+\frac{1}{\pi-1}$

$=\frac{0}{\pi}+\frac{1}{\pi-1}=\boxed{\frac{1}{\pi-1}}$

6. (15 points) Two functions $f(x)$ and $g(x)$ are graphed below.
Answer the following questions. (Short answer.)

(a) $f(3) = \boxed{1}$

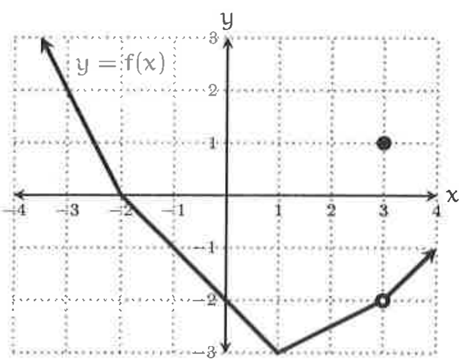
(b) $\lim_{x\to 2} g(x) = \boxed{3}$

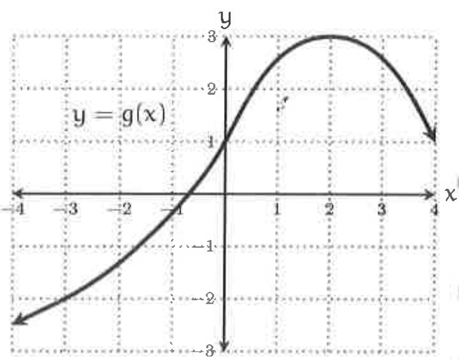
(c) $f\left(\lim_{x\to 2} g(x)\right) = f(3) = \boxed{1}$

(d) $\lim_{x\to 2} f(g(x)) = \boxed{-2}$

Because g(x) is approaching 3

(e) $\lim_{x\to -3} \frac{f(x)}{g(x)} = \frac{\lim_{x\to -3} f(x)}{\lim_{x\to -3} g(x)} = \frac{2}{-2} = \boxed{-1}$





Editorial Comment

According to a theorem in the text, if g(x) is continuous at 2 and f(x) is continuous at $\lim_{x\to 2} g(x) = 3$, then

$\lim_{x\to 2} f(g(x)) = f\left(\lim_{x\to 2} g(x)\right)$

They are <u>NOT</u> equal in (c) and (d) above. Note that f(x) is <u>NOT</u> continuous at $3 = \lim_{x\to 2} g(x)$