VCU

MATH 200

Calculus I

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Test 3

✍

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Name: Richard

Score: 100

Directions. Answer the questions in the space provided. Unless noted otherwise, you must show and explain your work to receive full credit. Put your final answer in a $\boxed{\text{box}}$ when appropriate.

This is a closed-book, closed-notes test. Calculators, computers, etc., are not used.

1. (30 points) Find the indefinite integrals.

(a) $\int (4x^5 + x + 2)\, dx =$ $4\frac{1}{6}x^6 + \frac{1}{2}x^2 + 2x + C$

$= \boxed{\frac{2x^6}{3} + \frac{x^2}{2} + 2x + C}$

(b) $\int \sqrt{x}\, dx =$ $\int x^{\frac{1}{2}} dx = \frac{1}{\frac{1}{2}+1} x^{\frac{1}{2}+1} + C$

$= \frac{1}{\frac{3}{2}} x^{\frac{3}{2}} + C = \boxed{\frac{2}{3}\sqrt{x}^{3} + C}$

(c) $\int \frac{1}{\sqrt{1-x^2}}\, dx =$ $\boxed{\sin^{-1}(x) + C}$

(d) $\int \sec(x)\tan(x) dx =$ $\boxed{\sec(x) + C}$

(e) $\int \frac{1}{x}\, dx =$ $\boxed{\ln|x| + C}$

2. (10 pts.)

(a) Is the following equation true or false? Explain.

$$\int x\cos(x)\,dx = x\sin(x) + \cos(x) + C$$

$\frac{d}{dx}$?

Let's check: $\frac{d}{dx}\left[x\sin(x) + \cos(x) + C\right]$

$= (1)\sin(x) + x\cos(x) - \sin(x) + 0$

$= x\cos(x).$

So yes. The equation is $\boxed{\text{true}}$

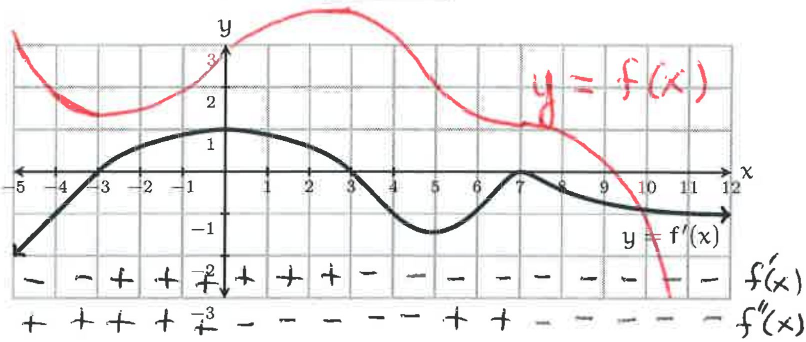
(b) If $f(x)$ and $g(x)$ are differentiable functions, then

$$\int (f'(x)g(x) + f(x)g'(x))\,dx = \boxed{f(x)\,g(x) + C}$$

because $\frac{d}{dx}\left[f(x)g(x) + C\right] =$

$f'(x)\,g(x) + f(x)\,g'(x)$

3. (10 pts.) The graph $y = f'(x)$ of **the derivative** of a function $f(x)$ is shown. Answer the questions about $f(x)$.



(a) State the intervals on which $f(x)$ increases.

$\boxed{(-3, 3)}$

(b) State the intervals on which $f(x)$ decreases.

$\boxed{(-\infty, -3) \cup (3, \infty)}$

(c) List all critical points of $f(x)$.

$\boxed{-3, \ 3, \ 7}$

(d) At which of its critical points does $f(x)$ have a local maximum?

$\boxed{3}$

(e) At which of its critical points does $f(x)$ have a local minimum?

$\boxed{-3}$

(f) State the intervals on which the function $f(x)$ is concave up.

$\boxed{(-\infty, 0) \cup (5, 7)}$

(g) State the intervals on which the function $f(x)$ is concave down.

$\boxed{(0, 5) \cup (7, \infty)}$

(h) Based on this information, sketch a possible graph of $f(x)$ on the coordinate axis above.

(sketched above in red)

4. (20 pts.) Find the limits.

(a) $\lim_{x \to 0} \dfrac{x^2}{\ln(\sec(x))} = \lim_{x \to 0} \dfrac{2x}{\dfrac{\sec(x)\tan(x)}{\sec(x)}}$

L.R.

form $\frac{0}{0}$

$$= \lim_{x \to 0} \frac{2x}{\tan x} = \lim_{x \to 0} \frac{2}{\sec^2(x)} = \frac{2}{\sec^2(0)}$$

form $\frac{0}{0}$

L.R.

$$= \frac{2}{1^2} = \boxed{2}$$

(b) $\lim_{x \to 0+} x \ln(x) = \lim_{x \to 0} \dfrac{\ln(x)}{\frac{1}{x}} = \lim_{x \to 0} \dfrac{\frac{1}{x}}{\frac{-1}{x^2}}$

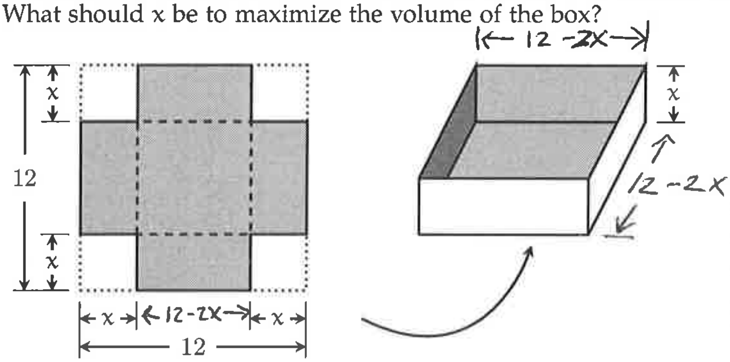
form $0 \cdot \infty$

form $\frac{\infty}{\infty}$

L.R.

$$= \lim_{x \to 0} \frac{1}{x} \cdot \frac{-x^2}{1} = \lim_{x \to 0} (-x) = \boxed{0}$$

5 (10 pts.) An open-top box is made from a 12 by 12 inch piece of cardboard by cutting a square from each corner, and folding up. What should x be to maximize the volume of the box?



Note: x can't be greater than 6 because that would make $12-2x$ negative

$$\text{Volume} = V(x) = lwh = (12-2x)^2 x$$

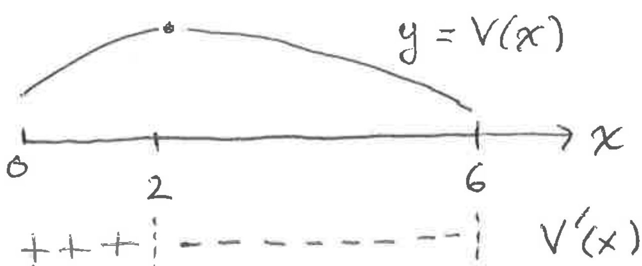
$$V(x) = (12-2x)^2 x$$

Maximize this on interval $[0,6]$

$$V'(x) = 2(12-2x)(-2)x + (12-2x)^2$$
$$= (12-2x)(-4x + (12-2x))$$
$$= (12-2x)(12-6x) = 0$$

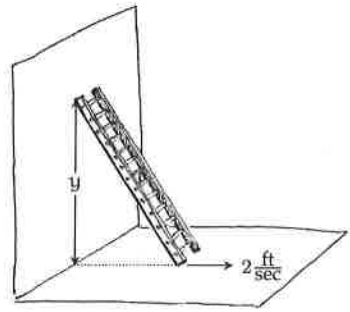
$x = 6$ (end point)

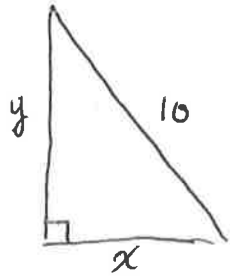
$x = 2$ critical point



Answer $\boxed{\text{Maximum volume at } x = 2''}$

6. (10 pts.) A 10-foot ladder is leaning against a wall, as illustrated. Its base slides away from the wall at a rate of 2 feet per second. At what rate is the height y changing when the base is 6 feet from the wall?





Know $\frac{dx}{dt} = 2$

Want $\frac{dy}{dt}$

when $x = 6$

$$x^2 + y^2 = 10^2$$

$$\frac{d}{dt}\left[x^2 + y^2\right] = \frac{d}{dt}\left[100\right]$$

$$2x\frac{dx}{dt} + 2y\frac{dy}{dt} = 0$$

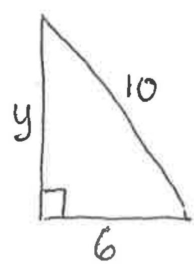
$$y\frac{dy}{dt} = -x\frac{dx}{dt}$$

$$\frac{dy}{dt} = -\frac{x}{y}\frac{dx}{dt}$$

$$= -\frac{6}{8}2$$

$$= -\frac{6}{4} = \boxed{-\frac{3}{2}\text{ feet/sec}}$$

Now calculate y at the instant $x=6$



$$y = \sqrt{10^2 - 6^2}$$
$$= \sqrt{100-36}$$
$$= \sqrt{64}$$
$$= 8$$

7. (10 pts.) Suppose $f(x)$ is a function for which $f'(x) = 2x + \cos(x)$ and $f(\pi) = 0$. Find $f(x)$.

$$f(x) = \int 2x + \cos(x)\, dx$$

$$= 2\frac{x^2}{2} + \sin(x) + C$$

$$f(x) = x^2 + \sin(x) + C$$

Now we just need to find C.

$$0 = f(\pi) = \pi^2 + \sin(\pi) + C$$

$$0 = \pi^2 + C$$

$$C = -\pi^2$$

Answer: $\boxed{f(x) = x^2 + \sin(x) - \pi^2}$