VCU

MATH 200

Calculus I

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Test 3

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Name: Richard

Score: ______________

Directions. Answer the questions in the space provided. Unless noted otherwise, you must show and explain your work to receive full credit. Put your final answer in a $\boxed{\text{box}}$ when appropriate.

This is a closed-book, closed-notes test. Calculators, computers, etc., are not used.

1. (30 points) Find the indefinite integrals.

(a) $\int (e^x + x^4 + 3)\,dx = \boxed{e^x + \frac{x^5}{5} + 3x + C}$

(b) $\int \frac{1}{\sqrt{x}}\,dx = \int x^{-\frac{1}{2}}\,dx = \frac{x^{-\frac{1}{2}+1}}{-\frac{1}{2}+1} + C$

$= \frac{x^{\frac{1}{2}}}{\frac{1}{2}} + C = \boxed{2\sqrt{x} + C}$

(c) $\int 5x^{-1}\,dx = 5\int \frac{1}{x}\,dx = \boxed{5\ln|x| + C}$

(d) $\int \sec^2(x)\,dx = \boxed{\tan(x) + C}$

(e) $\int \frac{1}{1+x^2}\,dx = \boxed{\tan^{-1}(x) + C}$

2. (10 pts.)

(a) Is the following equation true or false? Explain.

$$\int x\cos(x)\,dx = \frac{x^2}{2}\sin(x) + C$$

Check: $\frac{d}{dx}\left[\frac{x^2}{2}\sin(x) + C\right] =$

$x\sin(x) + \frac{x^2}{2}\cos(x) \neq x\cos(x)$

$\boxed{\text{No the equation is not true}}$

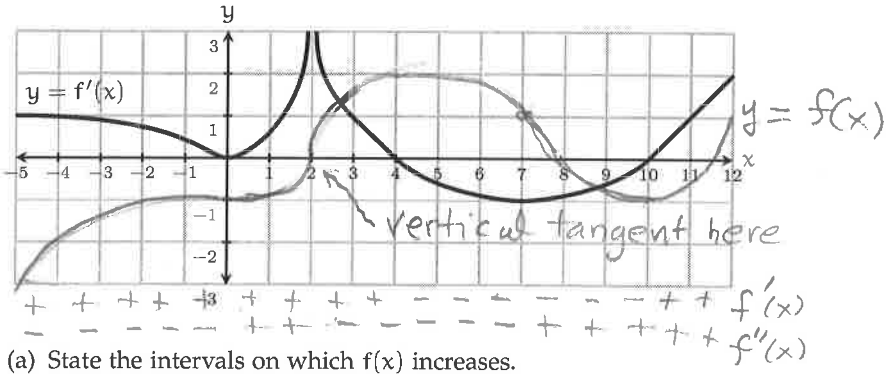
(b) If $f(x)$ and $g(x)$ are differentiable functions, then

$$\int f'(g(x))g'(x)\,dx = \boxed{f(g(x)) + C}$$

because $\frac{d}{dx}\left[f(g(x))\right]$

$= f'(g(x))g'(x)$ by the chain rule.

3. (15 pts.) The **derivative** $f'(x)$ of a function $f(x)$ is graphed below. Answer the questions about $f(x)$. (The domain of $f(x)$ is $(-5, 12)$.)



(a) State the intervals on which $f(x)$ increases.

$(-5, 4) \cup (10, 12)$

(b) State the intervals on which $f(x)$ decreases.

$(4, 10)$

(c) List all critical points of $f(x)$.

0, 2, 4, 10

(d) At which of its critical points does $f(x)$ have a local maximum?

$x = 4$

(e) At which of its critical points does $f(x)$ have a local minimum?

$x = 10$

(f) State the intervals on which the function $f(x)$ is concave up.

$(0, 2) \cup (7, 12)$

(g) State the intervals on which the function $f(x)$ is concave down.

$(-5, 0) \cup (2, 7)$

(h) Based on this information, sketch a possible graph of $f(x)$ on the coordinate axis above.

4. (20 pts.) Find the limits.

(a) $\displaystyle \lim_{x\to 0} \frac{8x^2}{\cos(x)-1} = \lim_{x\to 0} \frac{16x}{-\sin(x)} = \lim_{x\to 0} \frac{16}{-\cos(x)} = \boxed{-16}$

L.R. L.H.

form $\frac{0}{0}$ form $\frac{0}{0}$

form ∞^0 form $\frac{\infty}{\infty}$

(b) $\displaystyle \lim_{x\to\infty} (\ln(x))^{1/x} = \lim_{x\to\infty} e^{\ln\left((\ln(x))^{\frac{1}{x}}\right)}$

$\displaystyle = \lim_{x\to\infty} e^{\frac{1}{x}\ln(\ln(x))} = \lim_{x\to\infty} e^{\frac{\ln(\ln(x))}{x}}$

$\displaystyle = \lim_{x\to\infty} e^{\frac{\frac{1/x}{\ln(x)}}{1}} = \lim_{x\to\infty} e^{\frac{1}{x\ln(x)}} =$

$= e^0 = \boxed{1}$

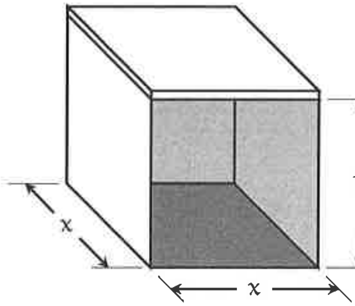
5 (15 pts.) You need to build a shed with an open front and square base (as illustrated), and containing a volume of 10,000 cubic feet. The cost of construction materials as follows:

Roof: \$10 per square foot;

Walls: \$8 per square foot;

Floor: \$5 per square foot.

What dimensions x and y will minimize the total cost of materials?



Volume $= x^2 y = 10000$

$$y = \frac{10000}{x^2}$$

Cost = (sides) + (top) + (floor)

$$= 3(8xy) + 10x^2 + 5x^2$$

$$= 24x\frac{10{,}000}{x^2} + 15x^2$$

$$C(x) = \frac{240000}{x} + 15x^2$$

Minimize this on $(0, \infty)$

$$C'(x) = -\frac{240000}{x^2} + 30x = 0$$

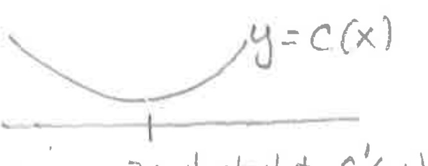
$$30x = \frac{240000}{x^2}$$

$$30x^3 = 240000$$

$$x^3 = \frac{240000}{30} = 8000$$

$$x = \sqrt[3]{8000} = 20$$

Critical point



Answer:

$x = 20'$

$$y = \frac{10000}{20^2} = \frac{10000}{400} = 25'$$

6. (10 pts.) Suppose $f(x)$ is a function for which $f'(x) = \frac{1}{2}\sec(x)\tan(x)$ and $f(0) = 1$. Find $f(x)$.

$$f(x) = \int \frac{1}{2} \sec(x) \tan(x) \, dx$$

$$f(x) = \frac{1}{2} \sec(x) + C$$

$$1 = f(0) = \frac{1}{2} \sec(0) + C$$

$$1 = \frac{1}{2} + C$$

$$C = \frac{1}{2}$$

$$\boxed{f(x) = \frac{1}{2} \sec(x) + \frac{1}{2}}$$