VCU

MATH 200

Calculus I

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Test 2

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Score: 100

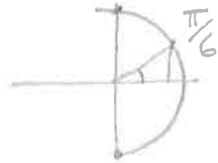
Directions. Answer the questions in the space provided. Unless noted otherwise, you must show and explain your work to receive full credit. Put your final answer in a $\boxed{\text{box}}$ when appropriate.

This is a closed-book, closed-notes test. Calculators, computers, etc., are not used.

1. (20 points) Warmup: short answer.

(a) $8^{4/3} = \sqrt[3]{8}^{\,4} = 2^4 = \boxed{16}$

(b) $\sin^{-1}(1/2) = \boxed{\frac{\pi}{6}}$



(c) $\ln\left(\sqrt[5]{e^7}\right) = \ln(e^{7/5}) = \boxed{\frac{7}{5}}$

(d) $e^{\ln(2)+\ln(3)} = e^{\ln(2)} e^{\ln(3)} = 2 \cdot 3 = \boxed{6}$

(e) $\log_{10}\left(\frac{1}{10}\right) = \boxed{-1}$

(f) $\frac{d}{dx}[\sin^{10}(x)] = \boxed{10 \sin^9(x) \cos(x)}$

(g) $\frac{d}{dx}[\sin(x^{10})] = \boxed{\cos(x^{10})\, 10x^9}$

(h) $\frac{d}{dx}[\sin(x)\, x^{10}] = \boxed{\cos(x)x^{10} + \sin(x)\, 10x^9}$

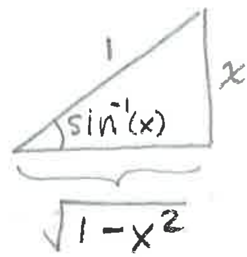
(i) $\frac{d}{dx}[\ln(x)] = \boxed{\frac{1}{x}}$

(j) $\frac{d}{dx}\left[\frac{1}{x}\right] = \frac{d}{dx}[x^{-1}] = -x^{-2} = \boxed{\frac{-1}{x^2}}$

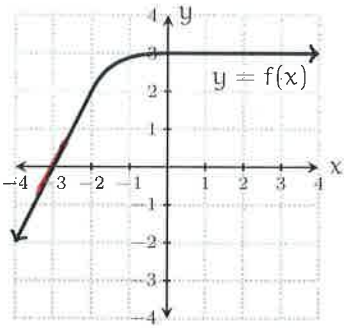
2. (5 points) Simplify: $\cos(\sin^{-1}(x)) =$

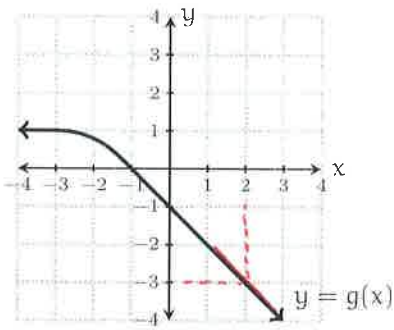
$= \frac{ADJ}{HYP} = \frac{\sqrt{1-x^2}}{1}$

$= \boxed{\sqrt{1-x^2}}$



3. (5 points) Two functions $f(x)$ and $g(x)$ are graphed below. Let $h(x) = f(g(x))$. Estimate $h'(2)$. Show your work.



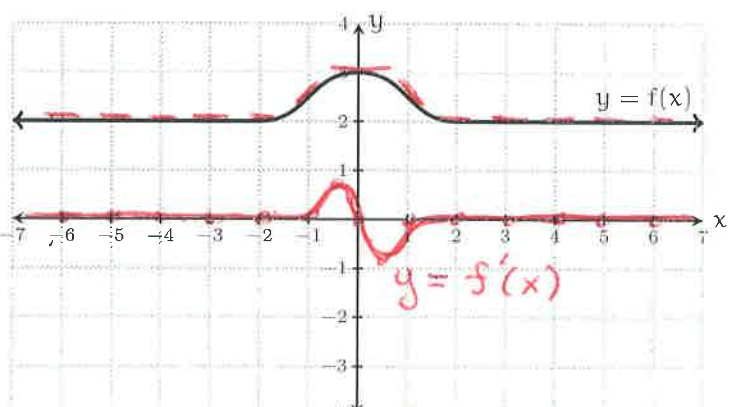


By chain rule, $h'(x) = f'(g(x))g'(x)$

Thus $h'(2) = f'(g(2))\,g'(2)$

$= f'(-3)\,g'(2) = 2(-1) = \boxed{-2}$

4. (5 points) A function $f(x)$ is graphed below. Using the same coordinate axis, sketch the graph of the derivative $f'(x)$.



5. (20 points) Find the following derivatives.

(a) $\frac{d}{dx}\left[\frac{x^2+5}{x+\sec(x)}\right] =$

$$\frac{2x(x+\sec(x)) - (x^2+5)(1+\sec(x)\tan(x))}{(x+\sec(x))^2}$$

(b) $\frac{d}{dx}[\tan^{-1}(5x)] =$ $\frac{5}{1+(5x)^2} = \boxed{\frac{5}{1+25x^2}}$

(c) $\frac{d}{dx}[\cos(\tan(x^3))] =$

$$-\sin(\tan(x^3))\frac{d}{dx}[\tan(x^3)]$$

$$= \boxed{-\sin(\tan(x^3))\sec^2(x^3)\,3x^2}$$

(d) $\frac{d}{dx}[\ln(xe^x)] =$ $\frac{1}{xe^x}\frac{d}{dx}[xe^x]$

$$= \frac{1}{xe^x}(1\cdot e^x + xe^x)$$

$$= \frac{e^x+xe^x}{xe^x} = \boxed{\frac{1+x}{x}}$$

6 (10 points) Find the inverse of the function $f(x) = e^{x^3+1}$.

$y = e^{x^3+1}$

$x = e^{y^3+1}$

$\ln(x) = \ln(e^{y^3+1})$

$\ln(x) = y^3 + 1$

using $\ln e^a = a$

$y^3 = \ln(x) - 1$

$y = \sqrt[3]{\ln(x)-1}$

Therefore $\boxed{f^{-1}(x) = \sqrt[3]{\ln(x) - 1}}$

7 (10 points) Suppose an object moves on a straight line in such a way that its distance from a fixed point at time t is $s(t) = t^3 - 9t^2 + 15t + 4$. Find the times t at which its velocity is 0.

velocity $= s'(t) = 3t^2 - 18t + 15 = 0$

$3(t^2 - 6t + 5) = 0$

$3(t-1)(t-5) = 0$

$t = 1$ $\quad$ $t = 5$

Answer:

$\boxed{\text{Velocity is zero at times } t = 1 \text{ and } t = 5}$

8. (5 points) State the limit definition of the derivative $f'(x)$ of a function $f(x)$.

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

9. (10 points) Suppose $f(x) = \sqrt{x}$. $= x^{\frac{1}{2}}$

Find the **equation** of the line tangent to the graph of $f(x)$ at the point $(9, 3)$.

$$f'(x) = \frac{1}{2} x^{-\frac{1}{2}} = \frac{1}{2x^{\frac{1}{2}}} = \frac{1}{2\sqrt{x}}$$

Slope of tangent is $f'(9) = \frac{1}{2\sqrt{9}} = \frac{1}{6}$

Equation of tangent line is

$y = \frac{1}{6} x + b$

Now find b using point (9, 3):

$3 = \frac{1}{6} \cdot 9 + b \rightsquigarrow b = 3 - \frac{3}{2} = \frac{6}{2} - \frac{3}{2} = \frac{3}{2}$

Answer: $\boxed{y = \frac{1}{6} x + \frac{3}{2}}$

10. (10 points) This question concerns the equation $x \sin(y) = y$.

(a) Use implicit differentiation to find $\frac{dy}{dx}$.

$$\frac{d}{dx}\left[x \sin y\right] = \frac{d}{dx}\left[y\right]$$

$$(1)\sin y + x\cos(y)\frac{dy}{dx} = \frac{dy}{dx}$$

$$x\cos(y)\frac{dy}{dx} - \frac{dy}{dx} = -\sin(y)$$

$$\frac{dy}{dx}\left(x\cos(y) - 1\right) = -\sin(y)$$

$$\boxed{\frac{dy}{dx} = \frac{-\sin(y)}{x\cos(y) - 1}}$$

(b) Use your answer from part (a) to find the slope of the tangent line to the graph of $x\sin(y) = y$ at the point $\left(\frac{\pi}{2}, \frac{\pi}{2}\right)$.

$$\left.\frac{dy}{dx}\right|_{(x,y) = \left(\frac{\pi}{2}, \frac{\pi}{2}\right)} = \frac{-\sin\left(\frac{\pi}{2}\right)}{\frac{\pi}{2}\cos\left(\frac{\pi}{2}\right) - 1}$$

$$= \frac{-1}{0 - 1} = \boxed{1}$$