VCU

MATH 200

Calculus I

R. Hammack

Test 2

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Name: Richard

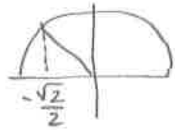
Score: 100

Directions. Answer the questions in the space provided. Unless noted otherwise, you must show and explain your work to receive full credit. Put your final answer in a $\boxed{\text{box}}$ when appropriate.

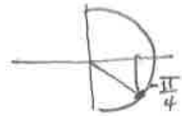
This is a closed-book, closed-notes test. Calculators, computers, etc., are not used.

1. (20 points) Warmup: short answer.

(a) $\cos^{-1}\left(-\frac{\sqrt{2}}{2}\right) = \boxed{\frac{3\pi}{4}}$



(b) $\tan^{-1}(-1) = \boxed{-\frac{\pi}{4}}$



(c) $\frac{d}{dx}\left[x^3 + 4\ln(x) + e^x\right] = \boxed{3x^2 + \frac{4}{x} + e^x}$

(d) $\frac{d}{dx}\left[\sec(x^5)\right] = \boxed{\sec(x^5)\tan(x^5)\, 5x^4}$

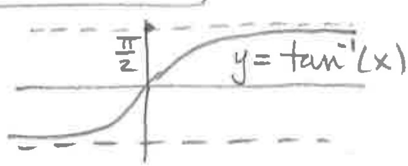
(e) $\frac{d}{dx}\left[(\sec(x))^5\right] =$ $5(\sec(x))^5 \sec(x)\tan(x)$

$= \boxed{5\sec^6(x)\tan(x)}$

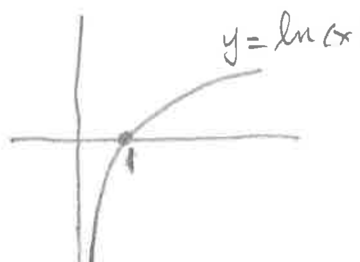
(f) $\frac{d}{dx}\left[e^x + \sin^{-1}(x)\right] = \boxed{e^x + \frac{1}{\sqrt{1-x^2}}}$

(g) $\frac{d}{dx}\left[\frac{\sin x}{x}\right] = \boxed{\frac{\cos(x)\,x - \sin(x)}{x^2}}$

(h) $\lim_{x\to\infty} \tan^{-1}(x) = \boxed{\frac{\pi}{2}}$

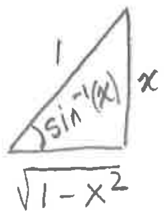


(i) $\lim_{x\to 0+} \ln(x) = \boxed{-\infty}$



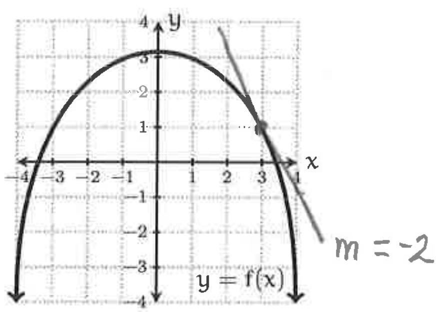
(j) $\lim_{x\to 1} \ln(x) = \boxed{0}$

2. (5 points) Simplify: $\tan(\sin^{-1}(x)) = \dfrac{\text{OPP}}{\text{ADJ}} = \boxed{\dfrac{x}{\sqrt{1-x^2}}}$

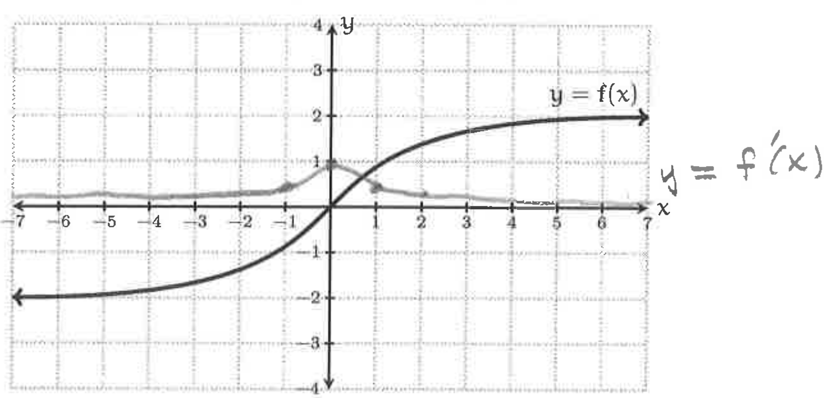


3. (5 points) For the function $f(x)$ below,

find $\lim\limits_{h\to 0} \dfrac{f(3+h)-f(3)}{h} = f'(3) = \left(\begin{array}{c}\text{slope of}\\ \text{tangent}\\ \text{at } x=3\end{array}\right) = \boxed{-2}$



4. (5 points) A function $f(x)$ is graphed below. Using the same coordinate axis, draw a rough sketch of the graph of $f'(x)$.



5. (10 points) Find the inverse of the function $f(x) = \dfrac{e^x}{1+e^x}$.

$$y = \frac{e^x}{1+e^x}$$

$$x = \frac{e^y}{1+e^y}$$

$$x(1+e^y) = e^y$$

$$x + xe^y = e^y$$

$$xe^y - e^y = -x$$

$$e^y(x-1) = -x$$

$$e^y = \frac{-x}{x-1}$$

$$\ln(e^y) = \ln\left(\frac{-x}{x-1}\right)$$

$$y = \ln\left(\frac{-x}{x-1}\right)$$

$$\boxed{f^{-1}(x) = \ln\left(\frac{-x}{x-1}\right)}$$

6 (10 points) Use logarithmic differentiation to find the derivative of $y = x^{\cos(x)}$.

$$\ln(y) = \ln(x^{\cos(x)})$$

$$\ln(y) = \cos(x)\ln(x)$$

$$\frac{d}{dx}[\ln(y)] = \frac{d}{dx}[\cos(x)\ln(x)]$$

$$\frac{1}{y}\frac{dy}{dx} = -\sin(x)\ln(x) + \cos(x)\frac{1}{x}$$

$$\frac{dy}{dx} = y\left(-\sin(x)\ln(x) + \frac{\cos(x)}{x}\right)$$

$$\boxed{\frac{dy}{dx} = x^{\cos(x)}\left(\frac{\cos(x)}{x} - \sin(x)\ln(x)\right)}$$

7 (20 points) Find the following derivatives.

(a) $\frac{d}{dx}\left[\sqrt{x^2+1}\right] = \frac{d}{dx}\left[(x^2+1)^{\frac{1}{2}}\right]$

$= \frac{1}{2}(x^2+1)^{-1/2}\,2x = \boxed{\dfrac{x}{\sqrt{x^2+1}}}$

(b) $\frac{d}{dx}\left[e^{\tan^{-1}(\pi x)}\right] = e^{\tan^{-1}(\pi x)}\,\frac{1}{1+(\pi x)^2}\,\pi$

$= \boxed{\dfrac{\pi e^{\tan^{-1}(\pi x)}}{1+\pi^2 x^2}}$

(c) $\frac{d}{dx}\left[\frac{\sqrt{x}\cos(x)}{x^3+1}\right] = \dfrac{\frac{d}{dx}\left[x^{\frac{1}{2}}\cos(x)\right](x^3+1) - \sqrt{x}\cos(x)\frac{d}{dx}\left[x^3+1\right]}{(x^3+1)^2}$

$\boxed{\dfrac{\left(\frac{1}{2}x^{-\frac{1}{2}}\cos(x) - x^{\frac{1}{2}}\sin(x)\right)(x^3+1) - \sqrt{x}\cos(x)\,3x^2}{(x^3+1)^2}}$

(d) $\frac{d}{dx}\left[\ln(x)\,e^x\right] = \boxed{\frac{1}{x}e^x + \ln x\, e^x}$

8. (5 points) $\lim_{h\to 0} \frac{\ln(x+h) - \ln(x)}{h} = \frac{d}{dx}\left[\ln(x)\right] = \boxed{\frac{1}{x}}$

definition of derivative

derivative rule

9. (10 points) Suppose $f(x) = x^3 - x + 2$.

Find the **equation** of the line tangent to the graph of $f(x)$ at the point $(2, 8)$.

$f'(x) = 3x^2 - 1$

slope $= f'(2) = 3 \cdot 2^2 - 1 = 11$

Point-slope form

$y - 8 = 11(x - 2)$

$y - 8 = 11x - 22$

$\boxed{y = 11x - 14}$

10. (10 points) This question concerns the equation $\ln(xy) = y + 1$.

$y = f(x)$

(a) Use implicit differentiation to find $\dfrac{dy}{dx}$.

$$\frac{d}{dx}\left[\ln(xy)\right] = \frac{d}{dx}\left[y + 1\right]$$

$$\frac{(1)y + xy'}{xy} = y' + 0$$

$$y + xy' = xyy'$$

$$xy' - xyy' = -y$$

$$y'(x - xy) = -y$$

$$y' = \frac{-y}{x - xy}$$

$$\boxed{\frac{dy}{dx} = \frac{-y}{x - xy}}$$

(b) Use your answer from part (a) to find the slope of the tangent line to the graph of $\ln(xy) = y + 1$ at the point $(-1, -1)$.

$$\left.\frac{dy}{dx}\right|_{(x,y)=(-1,-1)} = \frac{-(-1)}{-1 - (-1)(-1)} = \boxed{-\frac{1}{2}}$$