VCU

MATH 200

CALCULUS I

R. Hammack

TEST 1

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May 23, 2014

Name: Richard

Score: ____________

Directions. Answer the questions in the space provided. Unless noted otherwise, you must show and explain your work to receive full credit. Put your final answer in a $\boxed{\text{box}}$ when appropriate.

This is a closed-book, closed-notes test. Calculators, computers, etc., are not used.

1. (25 points) Warmup: short answer.

(a) $\tan\left(\frac{5\pi}{3}\right) = \dfrac{\sin\left(\frac{5\pi}{3}\right)}{\cos\left(\frac{5\pi}{3}\right)} = \dfrac{-\frac{\sqrt{3}}{2}}{\frac{1}{2}} = \boxed{-\sqrt{3}}$

(b) Describe the domain of $f(x) = \dfrac{x+1}{x\sqrt{x+5}}$.

Require $x+5>0$ (i.e. $x>-5$) and $x \neq 0$, so domain is $\boxed{(-5,0)\cup(0,\infty)}$

(c) Suppose $h(x) = \dfrac{\sin(\sqrt{x})}{\sqrt{x}}$.

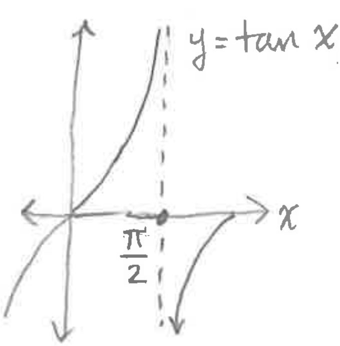
State functions $f(x)$ and $g(x)$ for which $h(x) = f \circ g(x)$.

$f(x) = \dfrac{\sin(x)}{x}$ and $g(x) = \sqrt{x}$

(d) $\lim\limits_{x\to 3}\left(\dfrac{x^2-1}{x^3}\right)^{\frac{2}{3}} = \left(\lim\limits_{x\to 3}\dfrac{x^2-1}{x^3}\right)^{2/3}$

$= \left(\dfrac{3^2-1}{3^3}\right)^{2/3} = \left(\dfrac{8}{27}\right)^{2/3} = \left(\sqrt[3]{\dfrac{8}{27}}\right)^2 = \left(\dfrac{2}{3}\right)^2 = \boxed{\dfrac{4}{9}}$

(e) $\lim\limits_{x\to \frac{\pi}{2}^+} \tan(x) = \boxed{-\infty}$



2. (15 points) Consider the equation $2\sin^2(x) = -\sin(x)$.
Find all solutions x of this equation for which $0 \leqslant x \leqslant 2\pi$.

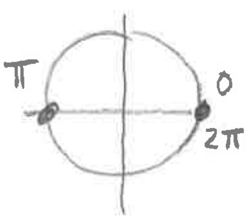
$$2(\sin(x))^2 = -\sin(x)$$

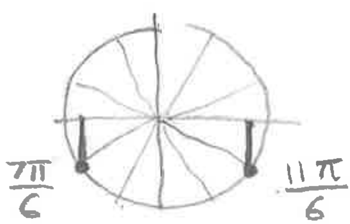
$$2(\sin(x))^2 + \sin(x) = 0$$

$$\sin(x)\left(2\sin(x) + 1\right) = 0$$

$$\sin(x) = 0$$

$$2\sin(x) + 1 = 0$$
$$2\sin(x) = -1$$
$$\sin(x) = -\frac{1}{2}$$





$$\boxed{\text{Answer: solutions are } x = 0,\ \pi,\ 2\pi,\ \frac{7\pi}{6},\ \frac{11\pi}{6}}$$

3. (15 points) Evaluate the following limits.

(a) $\lim_{x\to 2} \frac{\sin(2x-4)}{5x-10} = \lim_{x\to 2} \frac{1}{5}\frac{\sin(2x-4)}{x-2}$

$= \lim_{x\to 2} \frac{2\sin(2x-4)}{5\,2(x-2)}$

$= \lim_{x\to 2} \frac{2}{5}\frac{\sin(2x-4)}{2x-4} = \frac{2}{5}\cdot 1 = \boxed{\frac{2}{5}}$

(b) $\lim_{h\to 0} \frac{\sqrt{4+h}-2}{h} = \lim_{h\to 0} \frac{\sqrt{4+h}-2}{h}\frac{\sqrt{4+h}+2}{\sqrt{4+h}+2}$

$= \lim_{h\to 0} \frac{(4+h)+2\sqrt{4+h}-2\sqrt{4+h}-4}{h(\sqrt{4+h}+2)}$

$= \lim_{h\to 0} \frac{h}{h(\sqrt{4+h}+2)} = \lim_{h\to 0} \frac{1}{\sqrt{4+h}+2} = \frac{1}{\sqrt{4+0}+2}$

$= \boxed{\frac{1}{4}}$

(c) $\lim_{x\to 3} \frac{\frac{1}{x^2}-\frac{1}{9}}{x-3} = \lim_{x\to 3} \frac{\frac{9-x^2}{9x^2}}{\frac{x-3}{1}}$

$= \lim_{x\to 3} \frac{9-x^2}{9x^2}\frac{1}{x-3} = \lim_{x\to 3} \frac{\overset{(-1)}{\cancel{(3-x)}}(3+x)}{9x^2\cancel{(x-3)}}$

$= \lim_{x\to 3} \frac{-(3+x)}{9x^2} = \frac{-3+3}{9\cdot 3^2} = -\frac{6}{81}$

$= \boxed{-\frac{2}{27}}$

4. (15 points) Sketch the graph of any function that meets all of the following criteria.

1. $f(-1) = 3$

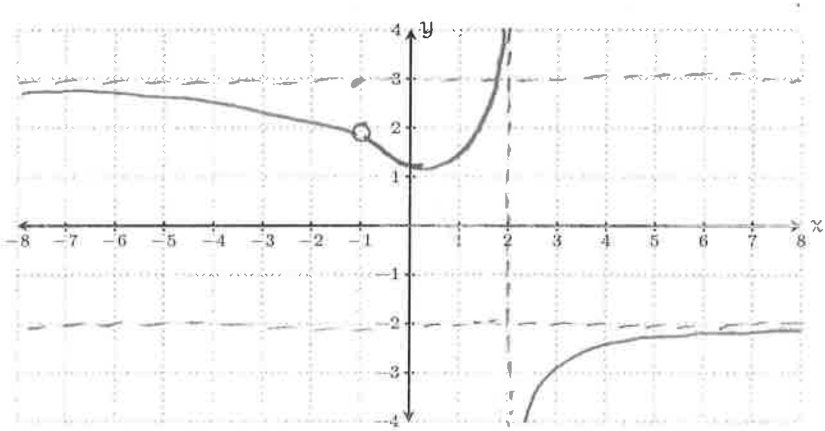
2. $\lim_{x \to \infty} f(x) = -2$

3. The line $y = 3$ is a horizontal asymptote

4. $\lim_{x \to 2^+} f(x) = -\infty$ and $\lim_{x \to 2^-} f(x) = \infty$

5. $\lim_{x \to -1} f(x) = 2$

6. $f(x)$ continuous at every x value except $x = -1$ and $x = 2$



5. (15 points) This question concerns the function $f(x) = \dfrac{15 - 12x - 3x^2}{50 - 2x^2}$.

(a) State the intervals on which $f(x)$ is continuous.

$$f(x) = \frac{-3(-5 + 4x + x^2)}{2(25 - x^2)} = \frac{-3}{2}\frac{(x^2+4x-5)}{(5-x)(5+x)}$$

$$= \frac{-3(x-1)(x+5)}{2(5-x)(5+x)}$$

Not continuous at $x = 5, -5$

Continuous on $\boxed{(-\infty,-5),\ (-5,5),\ (5,\infty)}$

(b) Find the horizontal asymptotes (if any).

From looking at coefficients of highest powers, $\lim_{x\to\infty} f(x) = \frac{-3}{-2} = \frac{3}{2}$

Thus $\boxed{\text{line } y = \frac{3}{2} \text{ is a H. A.}}$

(c) Find the vertical asymptotes (if any).

These could be located at either $x = 5$ or $x = -5$, where the denominator of $f(x)$ is zero.

Note by factoring above $f(x) = \dfrac{-3(x-1)}{2(5-x)}$

Test $x=5$: $\lim_{x\to 5^+} \dfrac{-3(x-1)}{2(5-x)} = \infty$

Thus $\boxed{\text{line } x = 5 \text{ is a VA}}$

Test $x=-5$ $\lim_{x\to -5^+} \dfrac{-3(x-1)}{2(5-x)} = \dfrac{-3(-5-1)}{2(5-(-5))} = \dfrac{18}{20}$

$\neq \pm\infty$ so no vertical asymptote here.

6. (15 points) Two functions $f(x)$ and $g(x)$ are graphed below. Answer the following questions.

(a) $\lim_{x \to 3} f(x) =$ $\boxed{2}$

(b) Find c if $\lim_{x \to c} f(x) = 0$. $\boxed{c = -1}$

(c) $\lim_{x \to -2} \frac{3f(x)g(x)}{\sqrt{12 + f(x)}} =$ $\frac{3(-3)(1)}{\sqrt{12 + -3}} = \frac{-9}{\sqrt{9}} = \boxed{-3}$

(d) $g \circ f(-2) =$ $g(f(-2)) = g(-3) = \boxed{\frac{1}{2}}$

(e) $\lim_{x \to 3} f(g(x)) =$ $f\left(\lim_{x \to 3} g(x)\right) = f(2) = \boxed{1}$