VCU

MATH 200

Calculus I

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Test 1

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Name: Richard

Score: 100

Directions. Answer the questions in the space provided. Unless noted otherwise, you must show and explain your work to receive full credit. Put your final answer in a $\boxed{\text{box}}$ when appropriate.

This is a closed-book, closed-notes test. Calculators, computers, etc., are not used.

1. (25 points) Warmup: quick answer.

(a) $25^{-0.5} = \dfrac{1}{25^{0.5}} = \dfrac{1}{25^{1/2}} = \dfrac{1}{\sqrt{25}} = \boxed{\dfrac{1}{5}}$

(b) State the domain of $f(x) = \sqrt{x+3} + \sqrt{7-x} + \dfrac{1}{x}$.

Need: $x + 3 \geq 0 \rightarrow x \geq -3$
$7 - x \geq 0 \rightarrow 7 \geq x$
$x \neq 0$

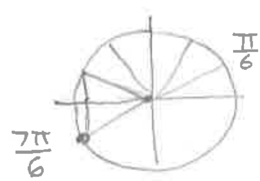
Therefore $\boxed{\text{domain is } [-3, 0) \cup (0, 7]}$

(c) If $f(x) = \sin(x)\cos(x)$ and $g(x) = \sqrt{x} + x$, then:

$f \circ g(x) = f(g(x)) = \boxed{\sin(\sqrt{x}+x)\cos(\sqrt{x}+x)}$

$g \circ f(x) = g(f(x)) = \boxed{\sqrt{\sin(x)\cos(x)} + \sin(x)\cos(x)}$

(d) $\cos \frac{7\pi}{6} = \boxed{-\dfrac{\sqrt{3}}{2}}$



(e) $\displaystyle \lim_{x\to\pi^+} \frac{\overbrace{\cos(x)}^{\text{approaches } -1}}{\underbrace{1+\cos(x)}_{\text{approaches 0 positive}}} = \boxed{-\infty}$

2. (10 points) Consider the equation $x\cos(x) = \cos(x)$.
Find all solutions x of this equation for which $0 \leqslant x \leqslant 2\pi$.

$$x\cos(x) = \cos(x)$$

$$x\cos(x) - \cos(x) = 0$$

$$\cos(x)(x-1) = 0$$

$\cos(x) = 0$

$x = \frac{\pi}{2}, \frac{3\pi}{2}$

$x - 1 = 0$

$x = 1$

Answer $\boxed{\text{solutions } x = \frac{\pi}{2},\ x = \frac{3\pi}{2},\ x = 1}$

3. (15 points) Evaluate the following limits.

(a) $\lim_{x \to 0} \frac{\sin(1 - \cos x)}{1 - \cos x} = \boxed{1}$

Note: as $x \to 0$, $1 - \cos(x)$ approaches 0, so this limit has form $\frac{\sin(\quad)}{\quad}$ with $\quad \to 0$. Because $\lim_{x \to 0} \frac{\sin(x)}{x} = 1$, the above limit equals 1.

(b) $\lim_{x \to 2} \frac{x^2 - 4}{x - 2} = \lim_{x \to 2} \frac{\cancel{(x-2)}(x+2)}{\cancel{x-2}}$

$= \lim_{x \to 2} (x+2) = 2 + 2 = \boxed{4}$

(c) $\lim_{x \to 1} \frac{\frac{1}{x} - 1}{x - 1} = \lim_{x \to 1} \frac{\frac{1-x}{x}}{\frac{x-1}{1}} = \lim_{x \to 1} \frac{1-x}{x} \frac{1}{x-1}$

$= \lim_{x \to 1} \frac{-(x-1)}{x(x-1)} = \lim_{x \to 1} \frac{-1}{x}$

$= \frac{-1}{1} = \boxed{-1}$

4. (15 points) Sketch the graph of any function that meets all of the following criteria.

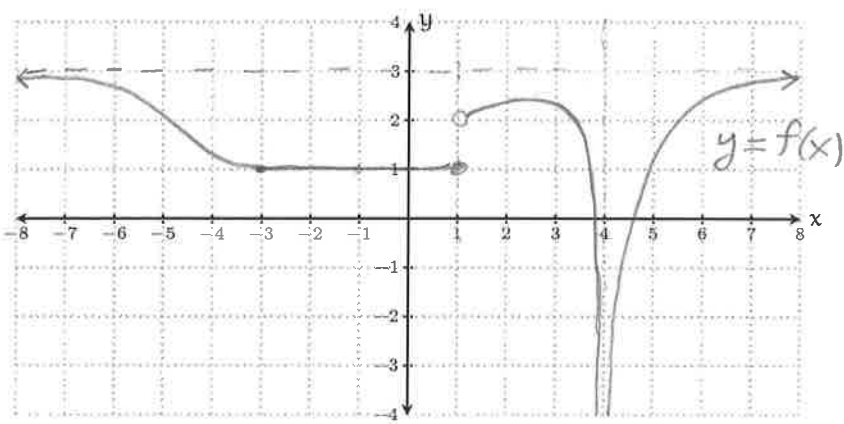
(a) The domain of $f(x)$ is all real numbers except $x = 4$

(b) $f(x)$ is continuous at all real numbers except $x = 1$ and $x = 4$

(c) $\lim_{x\to\infty} f(x) = 3$ and $\lim_{x\to 1+} f(x) = 2$

(d) The line $x = 4$ is a vertical asymptote

(e) $\lim_{x\to -3} f(x) = 1$



5. (20 points) Evaluate the following limits.

(a) $\lim\limits_{x\to 8} \dfrac{x-2}{\sqrt{x}(\sqrt{x}-\sqrt{2})} = \dfrac{8-2}{\sqrt{8}(\sqrt{8}-\sqrt{2})} = \dfrac{6}{2\sqrt{2}(2\sqrt{2}-\sqrt{2})}$

$= \dfrac{6}{2\sqrt{2}\sqrt{2}} = \dfrac{6}{4} = \boxed{\dfrac{3}{2}}$

(b) $\lim\limits_{x\to 2} \dfrac{x-2}{\sqrt{x}(\sqrt{x}-\sqrt{2})} = \lim\limits_{x\to 2} \dfrac{\cancel{(\sqrt{x}-\sqrt{2})}(\sqrt{x}+\sqrt{2})}{\sqrt{x}\,\cancel{(\sqrt{x}-2)}}$

$= \lim\limits_{x\to 2} \dfrac{\sqrt{x}+\sqrt{2}}{\sqrt{x}} = \dfrac{\sqrt{2}+\sqrt{2}}{\sqrt{2}} = \dfrac{2\sqrt{2}}{\sqrt{2}} = \boxed{2}$

(top approaches $\sqrt{2}$)

(c) $\lim\limits_{x\to 0+} \dfrac{x-2}{\sqrt{x}(\sqrt{x}-\sqrt{2})} = \lim\limits_{x\to 0^+} \dfrac{\sqrt{x}+\sqrt{2}}{\sqrt{x}} = \boxed{\infty}$

(factor as in part (b))

(Bottom approaches 0, and is positive)

(d) $\lim\limits_{x\to \infty} \dfrac{x-2}{\sqrt{x}(\sqrt{x}-\sqrt{2})} = \lim\limits_{x\to\infty} \dfrac{x-2}{x-\sqrt{2}\sqrt{x}}$

$= \lim\limits_{x\to\infty} \dfrac{x-2}{x-\sqrt{2}x^{1/2}} \cdot \dfrac{\frac{1}{x}}{\frac{1}{x}} = \lim\limits_{x\to\infty} \dfrac{1-\frac{2}{x}}{1-\frac{\sqrt{2}}{\sqrt{x}}}$

$= \dfrac{1-0}{1-0} = \boxed{1}$

6. (15 points) Two functions $f(x)$ and $g(x)$ are graphed below. Answer the following questions.

(a) $\lim_{x\to 3} f(x) =$ $\boxed{-2}$

(b) $f(3) =$ $\boxed{1}$

(c) $f\left(\lim_{x\to 1} g(x)\right) =$ $f(3) = \boxed{1}$

(d) $\lim_{x\to 1} f(g(x)) =$ $f\left(\begin{matrix}\text{quantity}\\ \text{approaching } -2\end{matrix}\right) = \boxed{3}$

(e) $\lim_{x\to 3} f(x)g(x) =$ $\left(\lim_{x\to 3} f(x)\right)\left(\lim_{x\to 3} g(x)\right) = (-2)(2)$

$= \boxed{-4}$