MATH 200

CALCULUS I

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TEST 3

April 12, 2013

Name: _____

Score: _____

Directions. Solve the following questions in the space provided. Unless noted otherwise, you must show your work to receive full credit. This is a closed-book, closed-notes test. Calculators, computers, etc., are not used. Put a your final answer in a box, where appropriate.

7. (10 pts.) Suppose $f(x)$ is a function for which $f'(x) = \dfrac{1}{x} + 3x$ and $f(1) = 5$. Find $f(x)$.

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1. (32 points) Find the indefinite integrals.

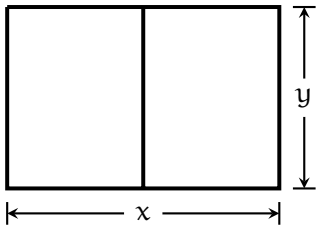
(a) $\displaystyle\int (5x + 3 + x^4)\,dx =$

(b) $\displaystyle\int \left(\frac{1}{x^2} + \sqrt{x}\right) dx =$

(c) $\displaystyle\int \frac{6}{\sqrt{1-x^2}}\,dx =$

(d) $\displaystyle\int 4\sin(3x)\,dx =$

2. (10 pts.) Suppose you have 120 feet of fencing material to enclose two rectangular regions, as illustrated. Find the dimensions x and y that maximize the total enclosed area.



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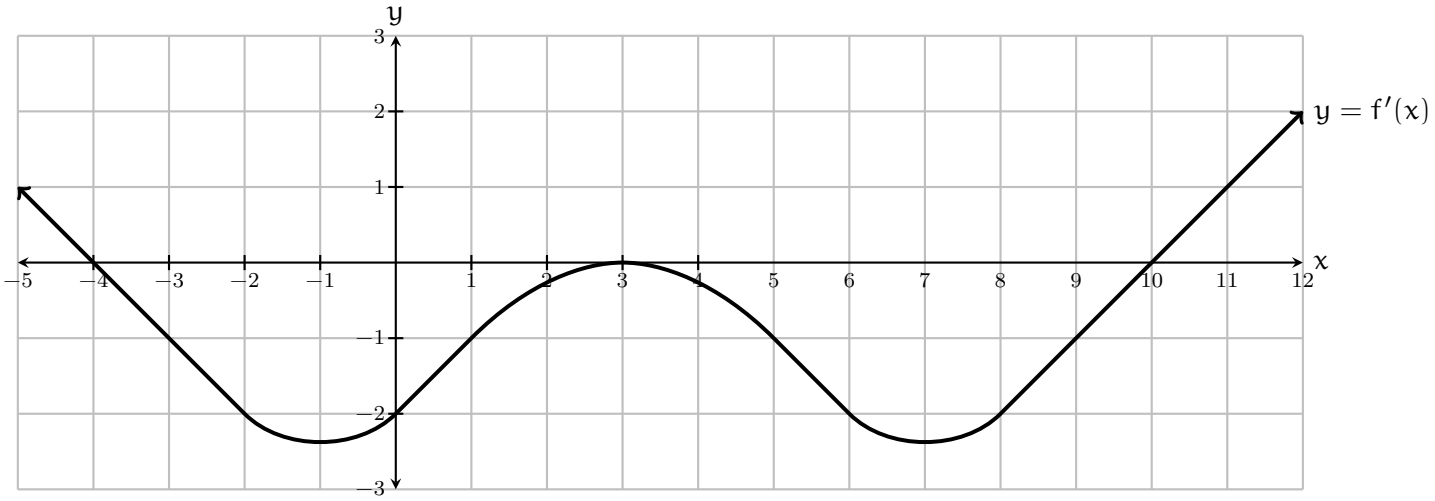
3. (10 pts.) The graph $y = f'(x)$ of **the derivative** of a function $f(x)$ is shown. Answer the questions about $f(x)$.



(a) State the intervals on which $f(x)$ increases.

(b) State the intervals on which $f(x)$ decreases.

(c) List all critical points of $f(x)$.

(d) At which of these critical points is there a local maximum?

(e) State the intervals on which the function $f(x)$ is concave up.

4. (20 pts.) Find the limits.

(a) $\lim_{x \to 0} \frac{3x^2}{\cos(x) - 1} =$

(b) $\lim_{x \to 0} (1 + x)^{\frac{1}{x}} =$

5. (8 pts.) Is the following equation true or false?

$$\int \frac{\sin\left(\frac{1}{x}\right)}{x^2} dx = \cos\left(\frac{1}{x}\right) + C$$

Explain.

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6. (10 pts.) A 13-foot ladder is leaning against a wall, as illustrated, when its base begins to slide away from the wall at a rate of 5 feet per second. At what rate is the angle θ changing when the base is 12 feet from the wall?