MATH 200

CALCULUS I

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TEST 3

April 12, 2013

Name: _____

Score: _____

Directions. Solve the following questions in the space provided. Unless noted otherwise, you must show your work to receive full credit. This is a closed-book, closed-notes test. Calculators, computers, etc., are not used. Put a your final answer in a box, where appropriate.

7. (10 pts.) Suppose $f(x)$ is a function for which $f'(x) = \sqrt{x} + 2$ and $f(4) = 7$. Find $f(x)$.

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1. (32 points) Find the indefinite integrals.

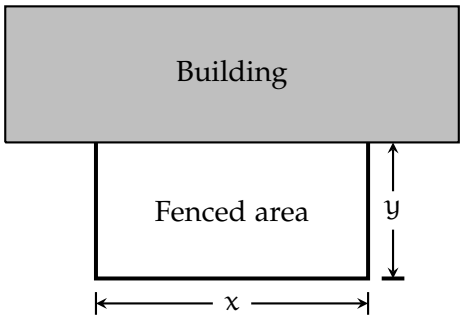
(a) $\int (x^5 + x + 1)\,dx =$

(b) $\int 4e^{3x}\,dx =$

(c) $\int \frac{5}{1+x^2}\,dx =$

(d) $\int \left(\frac{1}{x} + \cos(x)\right) dx =$

2. (10 pts.) Suppose you have 160 feet of fencing material to enclose a rectangular region. One side of the rectangle will border a building, so no fencing is required for that side. Find the dimensions x and y that maximize the fenced area.



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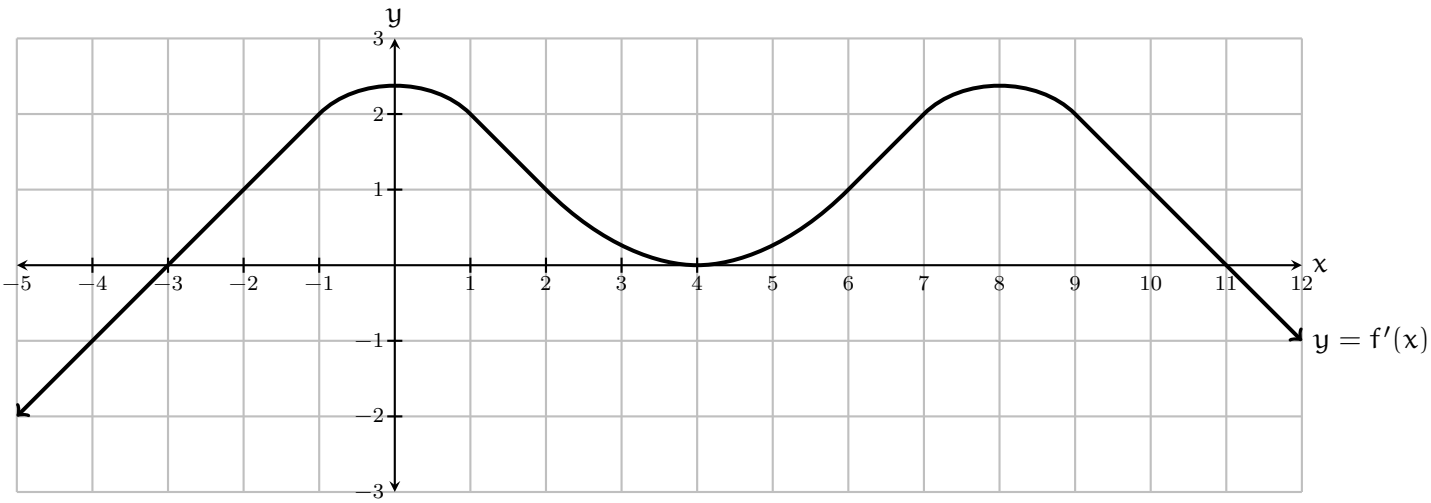
3. (10 pts.) The graph $y = f'(x)$ of **the derivative** of a function $f(x)$ is shown. Answer the questions about $f(x)$.



(a) State the intervals on which $f(x)$ increases.

(b) State the intervals on which $f(x)$ decreases.

(c) List all critical points of $f(x)$.

(d) At which of these critical points is there a local maximum?

(e) State the intervals on which the function $f(x)$ is concave up.

4. (20 pts.) Find the limits.

(a) $\lim_{x \to 0} \dfrac{1 - \cos x}{x^2} =$

(b) $\lim_{x \to 0^+} x^x =$

5. (8 pts.) Is the following equation true or false?

$$\int \left(\cos(x) \frac{1}{x} - \sin(x) \ln(x) \right) dx = \cos(x) \ln(x) + C$$

Explain.

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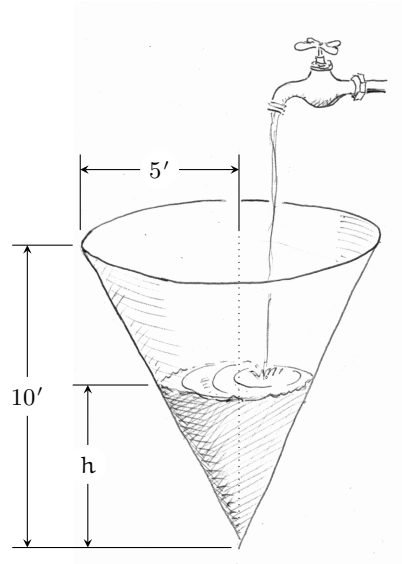
6. (10 pts.) Water flows into the conical tank (shown below) at a rate of 9 cubic feet per minute. How fast is the water level h rising when the water is 6 feet deep?



The volume of a cone of height h and radius r is $V = \frac{1}{3}\pi r^2 h$