MATH 200
CALCULUS I

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TEST 2

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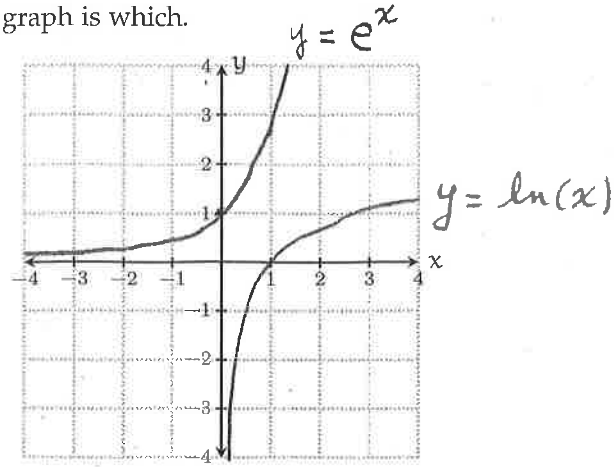
Name: Richard

Score: 100

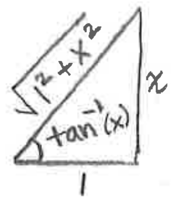
Directions. Solve the following questions in the space provided. Unless noted otherwise, you must show your work to receive full credit. This is a closed-book, closed-notes test. Calculators, computers, etc., are not used. Put a your final answer in a $\boxed{\text{box,}}$ where appropriate.

7. (10 points) Sketch the graph of both $y = \ln(x)$ and $y = e^x$ below. Be sure to indicate which graph is which.



8. (5 points) $\sin(\tan^{-1}(x)) = \frac{OPP}{HYP} = \boxed{\frac{x}{\sqrt{1+x^2}}}$

NOT THE DERIVATIVE



1. (20 points) Warmup: short answer.

(a) $\frac{d}{dx}[\sin(x) + \cos(x)] = \boxed{\cos(x) - \sin(x)}$

(b) $\frac{d}{dx}[\sin(x)\cos(x)] = \cos(x)\cos(x) - \sin(x)\sin(x)$
$= \boxed{\cos^2(x) - \sin^2(x)}$

(c) $\frac{d}{dx}[\sin(\cos(x))] = \cos(\cos(x))(-\sin(x))$
$= \boxed{-\cos(\cos(x))\sin(x)}$

(d) $\frac{d}{dx}[e^x] = \boxed{e^x}$

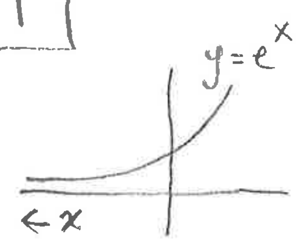
(e) $\frac{d}{dx}[x^e] = \boxed{e x^{e-1}}$

(f) $\ln(1/e) = \ln(e^{-1}) = \boxed{-1}$

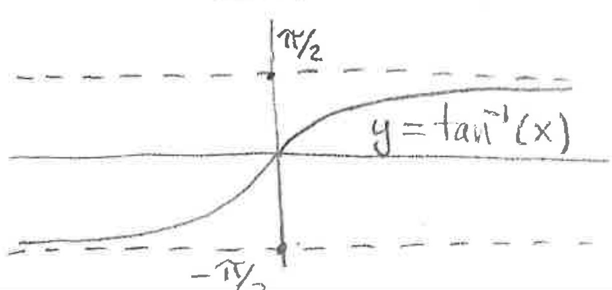
(g) $\sin^{-1}(1/2) = \boxed{\frac{\pi}{6}}$

(h) $e^{\cos(\pi/2)} = e^0 = \boxed{1}$

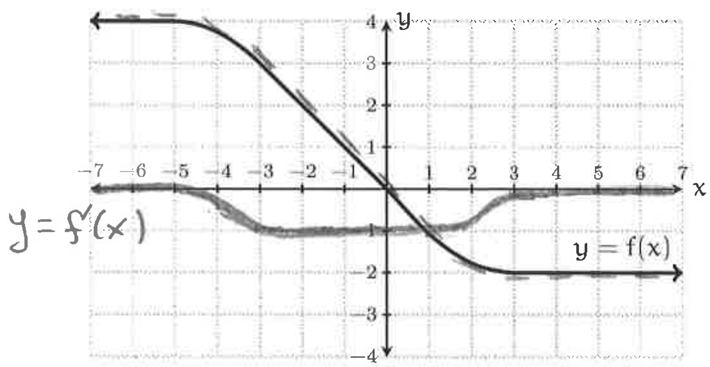
(i) $\lim_{x\to-\infty} e^x = \boxed{0}$



(j) $\lim_{x\to\infty} \tan^{-1}(x) = \boxed{\frac{\pi}{2}}$.



2. (10 points) Answer the following questions concerning the function $f(x)$ graphed below.



(a) Using the coordinate axis above, sketch the graph of the derivative $y = f'(x)$.

(b) Suppose $g(x) = (f(x))^3$. Find $g'(-2)$.

$$g'(x) = 3(f(x))^2 f'(x)$$

(by chain rule)

$$g'(-2) = 3(f(-2))^2 f'(-2)$$

$$= 3 \cdot 2^2 \cdot (-1)$$

$$= \boxed{-12}$$

3. (15 points) An object moving on a straight line is $s(t) = 2 + t + t^3$ feet from its starting point at time t seconds.

(a) What is the object's velocity at time t?

$$v(t) = s'(t) = \boxed{1 + 3t^2 \text{ ft/sec}}$$

(b) What is its acceleration at time t?

$$a(t) = v'(t) = \boxed{6t \text{ ft/sec/sec}}$$

(c) Find its velocity when its acceleration is 12 feet per second per second.

First find when $a(t) = 12$

$$6t = 12$$

$$t = 2 \text{ sec.}$$

Thus at time $t = 2$ sec, the acceleration is 12 ft/sec/sec. At this instant the velocity is $v(2) = 1 + 3 \cdot 2^2 = \boxed{13 \text{ ft/sec}}$

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4. (10 points) This problem concerns the functions $f(x) = x^3 - 3x$ and $g(x) = 3x^2 + 6x$. Find all x for which the tangent to $y = f(x)$ at $(x, f(x))$ is parallel to the tangent to $y = g(x)$ at $(x, g(x))$.

If the tangents are parallel, then slopes are equal, so we seek those x for which

$$f'(x) = g'(x)$$

$$3x^2 - 3 = 6x + 6$$

$$3x^2 - 6x - 9 = 0$$

$$3(x^2 - 2x - 3) = 0$$

$$3(x + 1)(x - 3) = 0$$

$x = -1$ $\qquad$ $x = 3$

Answer: For $\boxed{x = -1 \text{ and } x = 3}$ the tangent lines to $f(x)$ and $g(x)$ are parallel.

5. (20 points) Find the following derivatives.

(a) $\frac{d}{dx}[\ \ln(x) + \frac{1}{x} + \sqrt{x} + 3\ ] = \frac{d}{dx}\left[\ln(x) + x^{-1} + x^{\frac{1}{2}} + 3\right] = \frac{1}{x} - x^{-2} + \frac{1}{2}x^{-\frac{1}{2}}$

$= \boxed{\frac{1}{x} - \frac{1}{x^2} + \frac{1}{2\sqrt{x}}}$

(b) $\frac{d}{dx}\left[\left(\frac{x^2+5}{x+1}\right)^4\right] = \boxed{4\left(\frac{x^2+5}{x+1}\right)^3 \frac{2x(x+1) - (x^2+5)(1)}{(x+1)^2}}$

(c) $\frac{d}{dx}[\ \tan^{-1}(5x)\ ] = \frac{1}{1+(5x)^2}\frac{d}{dx}[5x] = \boxed{\frac{5}{1+25x^2}}$

(d) $\frac{d}{dx}[\ x\sec(e^{10x})\ ] = (1)\sec(e^{10x}) + x\sec(e^{10x})\tan(e^{10x})\frac{d}{dx}[e^{10x}]$

$= \sec(e^{10x}) + x\sec(e^{10x})\tan(e^{10x})\,e^{10x}\cdot 10$

$= \boxed{\sec(e^{10x}) + 10xe^{10x}\sec(e^{10x})\tan(e^{10x})}$

6. (10 points) This question concerns the equation $x^2 + xy + y^2 = 7$.

(a) Use implicit differentiation to find $\frac{dy}{dx}$.

$\frac{d}{dx}[x^2 + xy + y^2] = \frac{d}{dx}[7]$

$2x + (1)y + xy' + 2yy' = 0$

$xy' + 2yy' = -2x - y$

$y'(x+2y) = -2x - y$

$\frac{dy}{dx} = \boxed{y' = \frac{-2x-y}{x+2y}}$

(b) Use your answer from part (a) to find the **equation of the tangent line** to the graph of $x^2 + xy + y^2 = 7$ at the point $(2, -3)$.

$m = \left.\frac{dy}{dx}\right|_{(2,-3)} = \frac{-2\cdot 2 - (-3)}{2 + 2(-3)} = \frac{-1}{-4} = \frac{1}{4}$

Point-slope formula:

$y - y_0 = m(x - x_0)$

$y - (-3) = \frac{1}{4}(x - 2)$

$y + 3 = \frac{1}{4}x - \frac{1}{2}$

$y = \frac{1}{4}x - \frac{1}{2} - 3$

$\boxed{y = \frac{1}{4}x - \frac{7}{2}}$