MATH 200
CALCULUS I

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TEST 2

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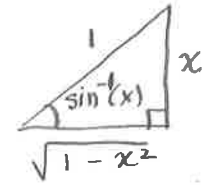
Name: Richard

Score: 100

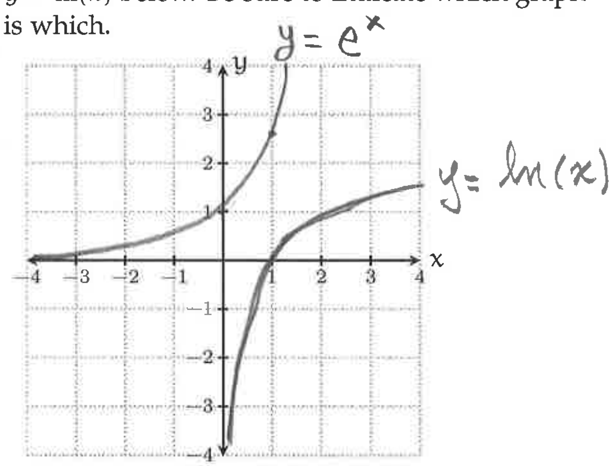
Directions. Solve the following questions in the space provided. Unless noted otherwise, you must show your work to receive full credit. This is a closed-book, closed-notes test. Calculators, computers, etc., are not used. Put a your final answer in a $\boxed{\text{box,}}$ where appropriate.

7. (5 points) Simplify: $\sec(\sin^{-1}(x)) = \dfrac{HYP}{ADJ} = \boxed{\dfrac{1}{\sqrt{1-x^2}}}$

NOT THE DERIVATIVE



8. (10 points) Sketch the graph of both $y = e^x$ **and** $y = \ln(x)$ below. Be sure to indicate which graph is which.



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1. (20 points) Warmup: short answer.

(a) $\dfrac{d}{dx}[\cos(x) + \ln(x)] = \boxed{-\sin(x) + \dfrac{1}{x}}$

(b) $\dfrac{d}{dx}[\cos(x)\ln(x)] = \boxed{-\sin(x)\ln(x) + \cos(x)\dfrac{1}{x}}$

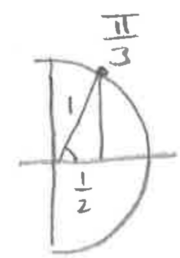
(c) $\dfrac{d}{dx}[\cos(\ln(x))] = \boxed{-\sin(\ln(x))\dfrac{1}{x}}$

(d) $\dfrac{d}{dx}[x^e] = \boxed{e x^{e-1}}$

(e) $\dfrac{d}{dx}[e^x] = \boxed{e^x}$

(f) $\ln(\sqrt{e}) = \ln(e^{1/2}) = \boxed{\dfrac{1}{2}}$

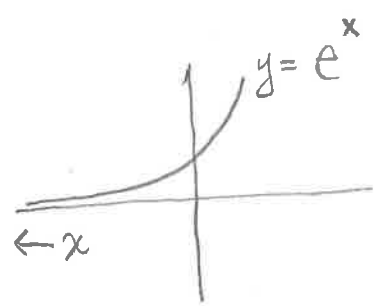
(g) $\cos^{-1}(1/2) = \boxed{\dfrac{\pi}{3}}$



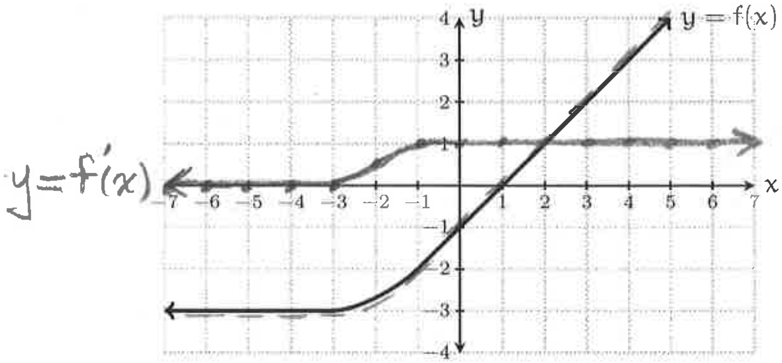
(h) $\ln(\sin(\pi/2)) = \ln(1) = \boxed{0}$

(i) $\lim_{x\to 1} \tan^{-1}(x) = \tan^{-1}(1) = \boxed{\dfrac{\pi}{4}}$

(j) $\lim_{x\to -\infty} e^x = \boxed{0}$



2. (10 points) Answer the following questions concerning the function $f(x)$ graphed below.



(a) Using the coordinate axis above, sketch the graph of the derivative $y = f'(x)$.

(b) Suppose $g(x) = x^2 f(x)$. Find $g'(3)$.

$g'(x) = 2x f(x) + x^2 f'(x)$

(by product rule)

$g'(3) = 2\cdot 3 \cdot f(3) + 3^2 f'(3)$

$= 2\cdot 3 \cdot 2 + 9 \cdot 1 = \boxed{21}$

3. (15 points) An object moving on a straight line is $s(t) = t^3 - 3t^2$ feet from its starting point at time t seconds.

(a) What is the object's velocity at time t?

$V(t) = S'(t) = \boxed{3t^2 - 6t \text{ ft/sec}}$

(b) What is its acceleration at time t?

$a(t) = V'(t) = \boxed{6t - 6 \text{ ft/sec/sec}}$

(c) Find its acceleration when its velocity is −3 feet per second.

Need t when $V(t) = -3$

$3t^2 - 6t = -3$

$3t^2 - 6t + 3 = 0$

$3(t^2 - 2t + 1) = 0$

$3(t-1)^2 = 0$

Solution : $t = 1$ sec. That is, velocity is −3 ft/sec when $t = 1$ sec.

Acceleration at this instant is $a(1) = 6\cdot 1 - 6$

$= \boxed{0 \text{ ft/sec/sec}}$

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4. (10 points) This problem concerns the functions $f(x) = x^2 + 2x^3$ and $g(x) = x^2 - 2x^3 + 48x$. Find all x for which the tangent to $y = f(x)$ at $(x, f(x))$ is parallel to the tangent to $y = g(x)$ at $(x, g(x))$.

The tangent lines are parallel when their slopes are equal, so we need to solve the following equation for x:

$f'(x) = g'(x)$

$2x + 6x^2 = 2x - 6x^2 + 48$

$12x^2 - 48 = 0$

$12(x^2 - 4) = 0$

$12(x-2)(x+2) = 0$

$x = 2 \qquad x = -2$

Answer : For $\boxed{x = 2 \text{ and } x = -2}$ the tangent lines to $y = f(x)$ and $y = g(x)$ are parallel.

5. (20 points) Find the following derivatives.

(a) $\frac{d}{dx}[\ \tan(x) + \frac{1}{x^2} + e^2 + 3\ ] = \frac{d}{dx}\left[\tan(x) + x^{-2} + e^2 + 3\right]$

$= \sec^2(x) - 2x^{-3} + 0 + 0 = \boxed{\sec^2(x) - \frac{2}{x^3}}$

(b) $\frac{d}{dx}\left[\sqrt{\frac{x^2+5}{x+1}}\ \right] = \frac{d}{dx}\left[\left(\frac{x^2+5}{x+1}\right)^{\frac{1}{2}}\right] = \frac{1}{2}\left(\frac{x^2+5}{x+1}\right)^{-\frac{1}{2}} \frac{2x(x+1)-(x^2+5)(1)}{(x+1)^2}$

$= \boxed{\frac{1}{2}\sqrt{\frac{x+1}{x^2+5}}\ \frac{x^2+2x-5}{(x+1)^2}}$

(c) $\frac{d}{dx}[\ \sin^{-1}(\pi x)\ ] = \frac{1}{\sqrt{1-(\pi x)^2}} \frac{d}{dx}[\pi x] = \boxed{\frac{\pi}{\sqrt{1-\pi^2 x^2}}}$

(d) $\frac{d}{dx}[\ xe^{\cos(3x)}\ ] = (1)e^{\cos(3x)} + x e^{\cos(3x)} \frac{d}{dx}[\cos(3x)]$

$= e^{\cos(3x)} + x e^{\cos(3x)}(-\sin(3x)\,3)$

$= \boxed{e^{\cos(3x)} - 3x\sin(3x)e^{\cos(3x)}}$

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6. (10 points) This question concerns the equation $xy^3 = xy + 6$.

(a) Use implicit differentiation to find $\frac{dy}{dx}$.

$\frac{d}{dx}[xy^3] = \frac{d}{dx}[xy+6]$

$(1)y^3 + x\,3y^2y' = (1)y + xy' + 0$

$3xy^2y' - xy' = y - y^3$

$y'(3xy^2 - x) = y - y^3$

$\frac{dy}{dx} = \boxed{y' = \frac{y-y^3}{3xy^2-x}}$

(b) Use your answer from part (a) to find the **equation of the tangent line** to the graph of $xy^3 = xy + 6$ at the point $(1, 2)$.

$m = \left.\frac{dy}{dx}\right|_{(1,2)} = \frac{2-2^3}{3\cdot1\cdot2^2-1} = \frac{-6}{11}$

Point slope formula

$y - y_0 = m(x - x_0)$

$y - 2 = -\frac{6}{11}(x-1)$

$y = -\frac{6}{11}x + \frac{6}{11} + 2$

$\boxed{y = -\frac{6}{11}x + \frac{28}{11}}$