VCU

MATH 200

CALCULUS I

R. Hammack

TEST 1

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September 18, 2015

Name: Richard

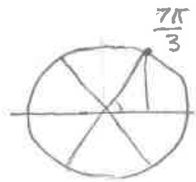
Score: 100

Directions. Answer the questions in the provided space. Unless noted otherwise, you must show and explain your work to receive full credit. Put your final answer in a $\boxed{\text{box}}$ when appropriate.

This is a closed-book, closed-notes test. Calculators, computers, etc., are not used. Please put all phones away.

1. (20 points) Warmup: short answer.

(a) $4^{3/2} = \sqrt{4}^3 = 2^3 = \boxed{8}$



(b) $\sin\left(\frac{7\pi}{3}\right) = \boxed{\frac{\sqrt{3}}{2}}$

(c) $\ln\left(\sqrt[5]{e}\right) = \ln e^{\frac{1}{5}} = e^{\square}\left(e^{\frac{1}{5}}\right) = \boxed{\frac{1}{5}}$

(d) $\ln\left(e^x\right) = \boxed{x}$

(e) $e^{\ln(3)+\ln(5)} = e^{\ln(3\cdot 5)} = e^{\ln(15)} = \boxed{15}$

(f) $\log_2(2) + \log_2\left(\frac{1}{8}\right) = 2^{\square}(2) + 2^{\square}\left(\frac{1}{8}\right) = 1 - 3 = \boxed{-2}$

(g) If $f(x) = \ln(x)$, then $f^{-1}(x) = \boxed{e^x}$

(h) $\sin^{-1}(-1) = \boxed{-\frac{\pi}{2}}$

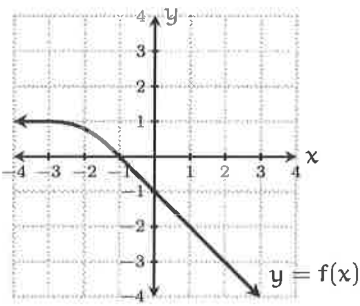
(i) $\sin\left(\sin^{-1}(0.3)\right) = \boxed{0.3}$

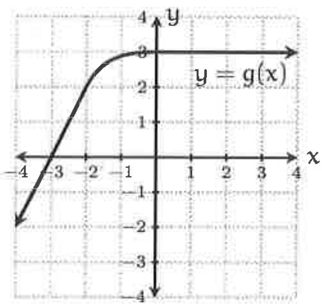
(j) $\lim\limits_{x\to\infty} \tan^{-1}(x) = \boxed{\frac{\pi}{2}}$ (see graph in # 3)

2. (10 points) For the functions $f(x)$ and $g(x)$ graphed below, find

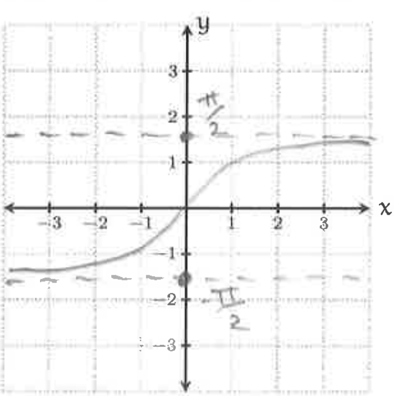
(a) $\lim_{x\to 1} f(x)g(x) = \left(\lim_{x\to 1} f(x)\right)\left(\lim_{x\to 1} g(x)\right) = (-2)(3) = \boxed{-6}$

(b) $\lim_{x\to -2} f(g(x)) = f\left(\lim_{x\to -2} g(x)\right) = f(2) = \boxed{-3}$





3. (5 points) Sketch the graph of $y = \tan^{-1}(x)$.



4. (20 points) Find the following limits.

(a) $\displaystyle\lim_{x\to 5}\frac{x-5}{x^2-25} = \lim_{x\to 5}\frac{\cancel{x-5}}{(\cancel{x-5})(x+5)} = \lim_{x\to 5}\frac{1}{x+5}$

$\displaystyle = \frac{1}{5+5} = \boxed{\frac{1}{10}}$

(b) $\displaystyle\lim_{x\to 2}\frac{\frac{1}{x}-\frac{1}{2}}{x-2} = \lim_{x\to 2}\frac{\frac{2}{2}\frac{1}{x}-\frac{1}{2}\frac{x}{x}}{\frac{x-2}{1}} = \lim_{x\to 2}\frac{\frac{2-x}{2x}}{\frac{x-2}{1}}$

$\displaystyle = \lim_{x\to 2}\frac{2-x}{2x}\,\frac{1}{2-x} = \lim_{x\to 2}\frac{-1}{2x}$

$\displaystyle = \frac{-1}{2\cdot 2} = \boxed{-\frac{1}{4}}$

(c) $\displaystyle\lim_{h\to 0}\frac{\sqrt{16+h}-4}{h} = \lim_{h\to 0}\frac{\sqrt{16+h}-4}{h}\,\frac{\sqrt{16+h}+4}{\sqrt{16+h}+4}$

$\displaystyle = \lim_{h\to 0}\frac{(16+h)-16}{h(\sqrt{16+h}+4)}$

$\displaystyle = \lim_{h\to 0}\frac{\cancel{h}}{\cancel{h}(\sqrt{16+h}+4)} = \frac{1}{\sqrt{16+0}+4} = \boxed{\frac{1}{8}}$

(d) $\displaystyle\lim_{x\to 1}\ln\left(\frac{x^2-1}{2x-2}\right) = \ln\left(\lim_{x\to 1}\frac{x^2-1}{2x-2}\right)$

$\displaystyle = \ln\left(\lim_{x\to 1}\frac{(x+1)(\cancel{x-1})}{2(\cancel{x-1})}\right)$

$\displaystyle = \ln\left(\lim_{x\to 1}\frac{x+1}{2}\right) = \ln\left(\frac{1+1}{2}\right)$

$\displaystyle = \ln\left(\frac{2}{2}\right) = \ln(1) = \boxed{0}$

5. (15 points) Sketch the graph of a function that meets all of the following criteria.

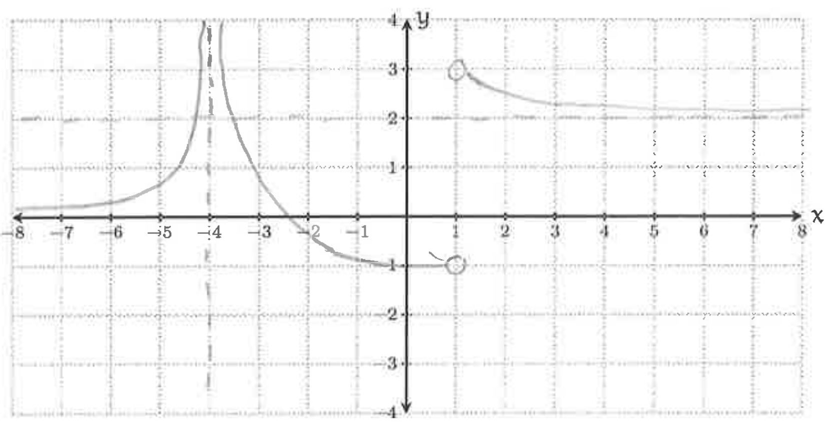
(a) The domain of $f(x)$ is all real numbers except $x = -4$ and $x = 1$

(b) $\lim_{x \to 1^+} f(x) = 3$, and $\lim_{x \to 1^-} f(x) = -1$

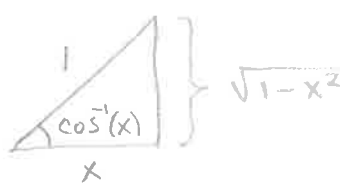
(c) $f(x)$ is continuous at all real numbers except $x = -4$ and $x = 1$

(d) $\lim_{x \to \infty} f(x) = 2$ and $\lim_{x \to -\infty} f(x) = 0$

(e) The line $x = -4$ is a vertical asymptote



6. (5 points) Simplify: $\tan(\cos^{-1}(x)) = \dfrac{\text{OPP}}{\text{ADJ}} = \boxed{\dfrac{\sqrt{1-x^2}}{x}}$



7. (5 points) Find the inverse of the function $f(x) = 2e^x - 1$.

$$y = 2e^x - 1$$

$$x = 2e^y - 1$$

$$x + 1 = 2e^y$$

$$\frac{x+1}{2} = e^y$$

$$\ln\left(\frac{x+1}{2}\right) = \ln(e^y)$$

$$\ln\left(\frac{x+1}{2}\right) = y$$

$$y = \ln\left(\frac{x+1}{2}\right)$$

$$\boxed{f^{-1}(x) = \ln\left(\frac{x+1}{2}\right)}$$

8. (10 points) Find all solutions of the equation $\sin^2(x) = \sin(x)$.

$$\sin^2(x) - \sin(x) = 0$$

$$\sin(x)(\sin(x) - 1) = 0$$

$\sin(x) = 0$ $\qquad$ $\sin(x) = 1$

Answer:

$$\boxed{\left.\begin{array}{l} x = k\pi \\ x = \frac{\pi}{2} + 2k\pi \end{array}\right\} \quad k = 0, \pm 1, \pm 2, \pm 3, \ldots}$$

9. (10 points) Find the horizontal and vertical asymptotes of the function $f(x) = \frac{2x^2 - 8}{x^2 + 3x + 2}$. $= \frac{2(x-2)(x+2)}{(x+1)(x+2)} = \frac{2(x-2)}{x+1}$

$$\lim_{x \to \infty} \frac{2x^2-8}{x^2+3x+2} = \frac{2}{1} = 2$$

$$\lim_{x \to -\infty} \frac{2x^2-8}{x^2+3x+2} = \frac{2}{1} = 2$$

$\boxed{\text{Line } y = 2 \text{ is H.A.}}$

To find the vertical asymptote(s), we first look at the x that make the denominator of f(x) equal to 0. From the factored form of f(x) (above) we see that $x = -1$ and $x = -2$ make the denominator 0. Thus -1 and -2 are the candidates for the locations of the V.A.

Test $x = -2$: $\lim_{x \to -2^+} f(x) = \lim_{x \to -2^+} \frac{2(x-2)}{x+1} = \frac{2(-2-2)}{-2+1} = 8$

Because $8 \neq \pm\infty$, no VA here.

Test $x = -1$: $\lim_{x \to -1^+} f(x) = \lim_{x \to -1} \frac{2(x-2)}{x+1} = -\infty$

approaches -6

approaches 0 positive

Because $\lim_{x \to -1^+} f(x) = -\infty$,

$\boxed{\text{the line } x = -1 \text{ is a V.A.}}$