VCU

MATH 200

Calculus I

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Test 1

☞

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Name: Richard

Score: 100

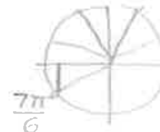
Directions. Answer the questions in the provided space. Unless noted otherwise, you must show and explain your work to receive full credit. Put your final answer in a $\boxed{\text{box}}$ when appropriate.

This is a closed-book, closed-notes test. Calculators, computers, etc., are not used. Please put all phones away.

1. (20 points) Warmup: short answer.

(a) $8^{2/3} = \sqrt[3]{8}^{\,2} = 2^2 = \boxed{4}$

(b) $\cos\left(\frac{7\pi}{6}\right) = \boxed{-\frac{\sqrt{3}}{2}}$



(c) $\ln\left(\sqrt{e^5}\right) = \ln\left(e^{5/2}\right) = e^{\square}\left(e^{\frac{5}{2}}\right) = \boxed{\frac{5}{2}}$

(d) $e^{\ln(x)} = \boxed{x}$

(e) $e^{\ln(4)+\ln(5)} = e^{\ln(4\cdot 5)} = 4\cdot 5 = \boxed{20}$

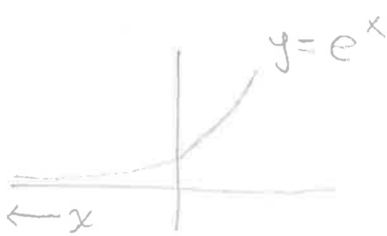
(f) $3\ln(2) + \ln\left(\frac{1}{8}\right) = \ln\left(2^3\right) + \ln\left(2^{-3}\right)$
$= 3\ln(2) - 3\ln(2) = \boxed{0}$

(g) If $f(x) = e^x$, then $f^{-1}(x) = \boxed{\ln(x)}$

(h) $\tan^{-1}(-1) = \boxed{-\frac{\pi}{4}}$

(i) $\sin\left(\sin^{-1}(0.5)\right) = \boxed{0.5}$

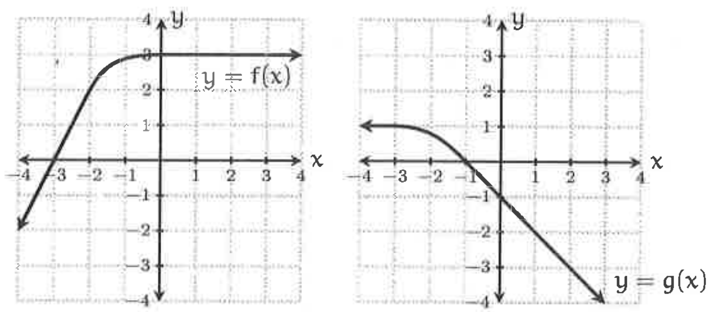
(j) $\lim\limits_{x\to-\infty} e^x = \boxed{0}$



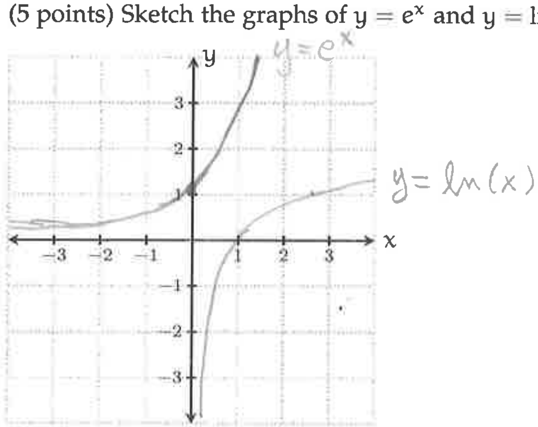
2. (10 points) For the functions $f(x)$ and $g(x)$ graphed below, find

(a) $\lim_{x\to 1} f(x)g(x) = \left(\lim_{x\to 1} f(x)\right)\left(\lim_{x\to 1} g(x)\right) = (3)(-2) = \boxed{-6}$

(b) $\lim_{x\to 1} f(g(x)) = f\left(\lim_{x\to 1} g(x)\right) = f(-2) = \boxed{2}$



3. (5 points) Sketch the graphs of $y = e^x$ and $y = \ln(x)$.



4. (20 points) Find the following limits.

(a) $\lim_{x\to 5} \frac{x^2-25}{x-5} = \lim_{x\to 5} \frac{\cancel{(x-5)}(x+5)}{\cancel{x-5}} = \lim_{x\to 5}(x+5)$

$= 5+5 = \boxed{10}$

(b) $\lim_{x\to 9} \frac{\sqrt{x}-3}{x-9} = \lim_{x\to 9} \frac{\cancel{\sqrt{x}-3}}{\cancel{(\sqrt{x}-3)}(\sqrt{x}+3)} = \lim_{x\to 9} \frac{1}{\sqrt{x}+3}$

$= \frac{1}{\sqrt{9}+3} = \frac{1}{3+3} = \boxed{\frac{1}{6}}$

(c) $\lim_{h\to 0} \frac{\frac{1}{2+h}-\frac{1}{2}}{h} = \lim_{h\to 0} \frac{\frac{2}{2}\frac{1}{2+h} - \frac{1}{2}\frac{2+h}{2+h}}{h}$

$= \lim_{h\to 0} \frac{\frac{2-(2+h)}{2(2+h)}}{\frac{h}{1}} = \lim_{h\to 0} \frac{-h}{2(2+h)} \frac{1}{h}$

$= \lim_{h\to 0} \frac{-1}{2(2+h)} = \frac{-1}{2\cdot 2} = \boxed{\frac{-1}{4}}$

(d) $\lim_{x\to 0} \sin\left(\frac{\pi x + x^2}{4x}\right) =$

$= \sin\left(\lim_{x\to 0} \frac{x(\pi+x)}{4x}\right) = \sin\left(\lim_{x\to 0} \frac{\pi+x}{4}\right)$

$= \sin\left(\frac{\pi+0}{4}\right) = \sin\left(\frac{\pi}{4}\right) = \boxed{\frac{\sqrt{2}}{2}}$

5. (15 points) Sketch the graph of a function that meets all of the following criteria.

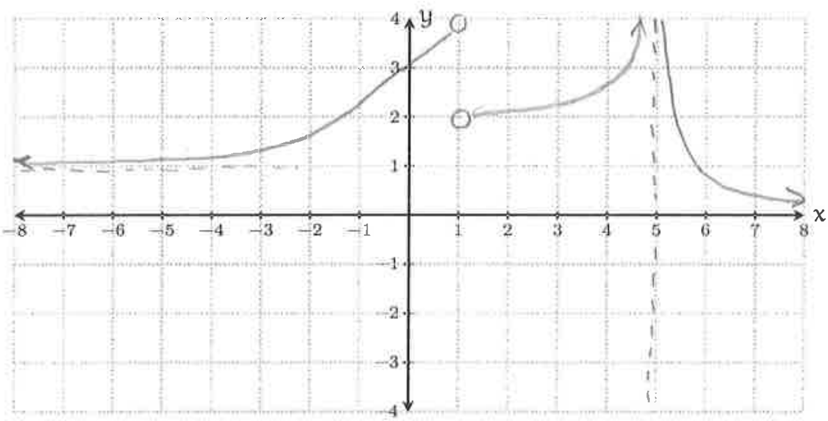
(a) The domain of $f(x)$ is all real numbers except $x = 1$ and $x = 5$

(b) $\lim_{x\to 1+} f(x) = 2$, and $\lim_{x\to 1-} f(x) = 4$

(c) $f(x)$ is continuous at all real numbers except $x = 1$ and $x = 5$

(d) $\lim_{x\to\infty} f(x) = 0$ and $\lim_{x\to -\infty} f(x) = 1$

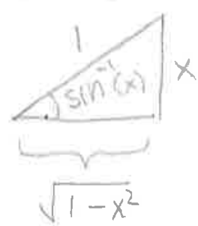
(e) The line $x = 5$ is a vertical asymptote



6. (5 points) Simplify: $\cos(\sin^{-1}(x)) =$

$$\frac{\text{ADJ}}{\text{HYP}} = \frac{\sqrt{1-x^2}}{1}$$

$$= \boxed{\sqrt{1-x^2}}$$



7. (5 points) Find the inverse of the function $f(x) = e^{2x} + 1$.

$$y = e^{2x} + 1$$

$$x = e^{2y} + 1$$

$$x - 1 = e^{2y}$$

$$\ln(x-1) = \ln(e^{2y})$$

$$\ln(x-1) = 2y$$

$$y = \frac{1}{2}\ln(x-1)$$

$$y = \ln(\sqrt{x-1})$$

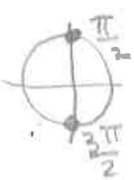
$$\therefore \boxed{f^{-1}(x) = \ln(\sqrt{x-1})}$$

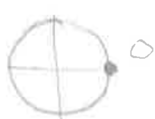
8. (10 points) Find all solutions of the equation $\cos^2(x) = \cos(x)$.

$$\cos^2(x) - \cos(x) = 0$$

$$\cos(x)\left(\cos(x) - 1\right) = 0$$

$\cos(x) = 0$ $\qquad\qquad$ $\cos(x) = 1$





Answer

$$\left.\begin{aligned} x &= \frac{\pi}{2} + k\pi \\ x &= 2k\pi \end{aligned}\right\} k = 0, \pm 1, \pm 2 \ldots$$

9. (10 points) Find the horizontal and vertical asymptotes of the function $f(x) = \frac{x^2+x-6}{2x^2-18}$. $= \frac{(x+3)(x-2)}{2(x-3)(x+3)} = \frac{x-2}{2(x-3)}$

$$\lim_{x\to\infty} \frac{x^2+x-6}{2x^2-18} = \frac{1}{2}$$

$$\lim_{x\to-\infty} \frac{x^2+x-6}{2x^2-18} = \frac{1}{2}$$

$\boxed{\text{line } y = \frac{1}{2} \text{ is H.A.}}$

Denominator of $f(x)$ is 0 when $x = \pm 3$

These are the candidates for locations of vertical asymptotes. Checking:

Check $x = -3$ $\quad \lim_{x\to -3} f(x) = \lim_{x\to-3} \frac{x-2}{2(x-3)}$

$$= \frac{-3-2}{2(-3-3)} = \frac{5}{12}$$

(no asymptote at $x = -3$)

Check $x = 3$ $\quad \lim_{x\to 3^+} f(x) = \lim_{x\to 3^+} \frac{x-2}{2(x-3)} = \infty$

$\boxed{\text{Therefore line } x = 3 \text{ is a VA}}$