MATH 200
Chapter 3 Summary and Test Review

R. Hammack
A. Hoeft

THIS study guide summarizes the main topics of Chapter 3, topics that you should expect will be represented on the test. But please remember that merely remembering these facts is not sufficient preparation for the test; you must work lots of exercises for practice. Please see the Exercise list on the MATH 200 web page.

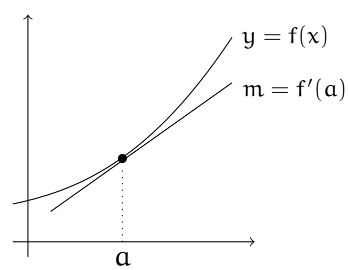
The Derivative and Its Interpretations

Chapter 3 deals entirely with a mathematical concept called a *derivative* of a function.

> The **derivative** of a function $f(x)$ is another function $f'(x)$ defined as
>
> $$\begin{aligned} f'(x) &= \lim_{h\to 0} \frac{f(x+h)-f(x)}{h} \\ &= \lim_{z\to x} \frac{f(z)-f(x)}{z-x}. \end{aligned}$$
>
> **Notation:** The derivative of $y = f(x)$ can be denoted in various ways, including $f'(x) = y' = \frac{dy}{dx} = \frac{d}{dx}[f(x)]$.

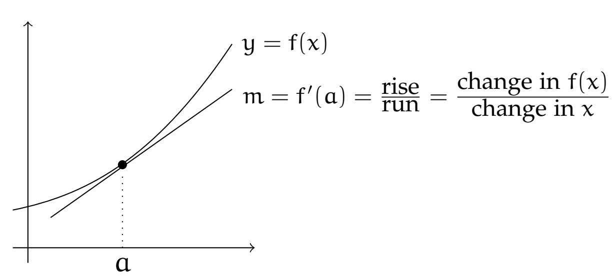
Because of its limit definition, the derivative has the following basic interpretations (see Section 3.6):

$f'(a)$ = **Slope** of the line tangent to the graph of $y = f(x)$ at the point $x = a$.



$f'(a)$ = **Velocity** at time $t = a$ of an object whose position on a straight line at time t is $f(t)$.

$f'(a)$ = **Instantaneous rate of change** of the quantity $f(x)$ with respect to x at $x = a$.



It is important to realize that, although we rarely use the limit definition to compute derivatives, the above interpretations come from the limit definition of $f'(x)$. In other words, it is because of its limit definition that the derivative has a *meaning*. Since we have short-cut rules for computing most derivatives, it is easy to lose sight of the importance of the limit. But without it, the derivative would have no meaning, and there would be no purpose in doing calculus.

Derivative Rules

The following short-cut rules allow us to compute derivatives of certain functions without a limit. You are expected to remember and internalize each rule on this page.

Derivative of a Constant:	$\frac{d}{dx}\left[\, c \,\right]$	$=$	0
Derivative of Identity:	$\frac{d}{dx}\left[\, x \,\right]$	$=$	1
Constant Multiple Rule:	$\frac{d}{dx}\left[\, cf(x) \,\right]$	$=$	$cf'(x)$
Sum/Difference Rule:	$\frac{d}{dx}\left[f(x) \pm g(x)\right]$	$=$	$f'(x) \pm g'(x)$
Product Rule:	$\frac{d}{dx}\left[f(x)g(x)\right]$	$=$	$f'(x)g(x) + f(x)g'(x)$
Quotient Rule:	$\frac{d}{dx}\left[\frac{f(x)}{g(x)}\right]$	$=$	$\frac{f'(x)g(x) - f(x)g'(x)}{(g(x))^2}$
Inverse Rule:	$\frac{d}{dx}\left[\, f^{-1}(x) \,\right]$	$=$	$\frac{1}{f'\left(f^{-1}(x)\right)}$ (Not often used.)
Chain Rule:	$\frac{d}{dx}\left[\, f\big(g(x)\big) \,\right]$	$=$	$f'(g(x))\, g'(x)$

Additional rules are listed on the left of the following table. The functions listed here can be composed with a second function $g(x)$, and the chain rule can be applied to find the derivative of the composition. For each rule on the left, the corresponding derivative of the composition is indicated on the right.

	Rule	**Chain Rule Generalization**
Power Rule:	$\frac{d}{dx}\left[\, x^n \,\right] = nx^{n-1}$	$\frac{d}{dx}\left[\, \big(g(x)\big)^n \,\right] = n\big(g(x)\big)^{n-1} g'(x)$
Natural Exp. Rule:	$\frac{d}{dx}\left[\, e^x \,\right] = e^x$	$\frac{d}{dx}\left[\, e^{g(x)} \,\right] = e^{g(x)} g'(x)$
Natural Log Rule:	$\frac{d}{dx}\left[\, \ln(x) \,\right] = \frac{1}{x}$	$\frac{d}{dx}\left[\, \ln\big(g(x)\big) \,\right] = \frac{1}{g(x)} g'(x)$
	$\frac{d}{dx}[\sin(x)] = \cos(x)$	$\frac{d}{dx}\left[\sin\big(g(x)\big)\right] = \cos\big(g(x)\big)\, g'(x)$
	$\frac{d}{dx}[\cos(x)] = -\sin(x)$	$\frac{d}{dx}\left[\cos\big(g(x)\big)\right] = -\sin\big(g(x)\big)\, g'(x)$
Trig Rules	$\frac{d}{dx}[\tan(x)] = \sec^2(x)$	$\frac{d}{dx}\left[\tan\big(g(x)\big)\right] = \sec^2\big(g(x)\big)\, g'(x)$
	$\frac{d}{dx}[\cot(x)] = -\csc^2(x)$	$\frac{d}{dx}\left[\cot\big(g(x)\big)\right] = -\csc^2\big(g(x)\big)\, g'(x)$
	$\frac{d}{dx}[\sec(x)] = \sec(x)\tan(x)$	$\frac{d}{dx}\left[\sec\big(g(x)\big)\right] = \sec\big(g(x)\big)\tan\big(g(x)\big)\, g'(x)$
	$\frac{d}{dx}[\csc(x)] = -\csc(x)\cot(x)$	$\frac{d}{dx}\left[\csc\big(g(x)\big)\right] = -\csc\big(g(x)\big)\cot\big(g(x)\big)\, g'(x)$
	$\frac{d}{dx}\left[\sin^{-1}(x)\right] = \frac{1}{\sqrt{1-x^2}}$	$\frac{d}{dx}\left[\sin^{-1}\big(g(x)\big)\right] = \frac{1}{\sqrt{1-(g(x))^2}}\, g'(x)$
Inverse Trig Rules	$\frac{d}{dx}\left[\cos^{-1}(x)\right] = \frac{-1}{\sqrt{1-x^2}}$	$\frac{d}{dx}\left[\cos^{-1}\big(g(x)\big)\right] = \frac{-1}{\sqrt{1-(g(x))^2}}\, g'(x)$
	$\frac{d}{dx}\left[\tan^{-1}(x)\right] = \frac{1}{1+x^2}$	$\frac{d}{dx}\left[\tan^{-1}\big(g(x)\big)\right] = \frac{1}{1+(g(x))^2}\, g'(x)$
	$\frac{d}{dx}\left[\sec^{-1}(x)\right] = \frac{1}{\lvert x\rvert\sqrt{x^2-1}}$	$\frac{d}{dx}\left[\tan^{-1}\big(g(x)\big)\right] = \frac{1}{\lvert g(x)\rvert\sqrt{(g(x))^2-1}}\, g'(x)$

Secondary Rules

The following rules are listed for completeness. You may occasionally find them useful on homework (or in future classes), but they will not be a focus of the test. In calculus, we tend to work almost exclusively with the base e, so the derivatives of exponential or logarithm functions to a base $a \neq e$ listed below are rarely necessary. Also, we will find that applications requiring an inverse trig function can almost always be phrased without using $\cot^{-1}(x)$ or $\csc^{-1}(x)$, so you do not need to know those two derivatives, listed below.

Rule	**Chain Rule Generalization**
$\dfrac{d}{dx}[a^x] = a^x \ln(a)$	$\dfrac{d}{dx}\left[a^{g(x)}\right] = a^{g(x)} \ln(a)\, g'(x)$
$\dfrac{d}{dx}[\log_a(x)] = \dfrac{1}{x\ln(a)}$	$\dfrac{d}{dx}\left[\log_a\left(g(x)\right)\right] = \dfrac{1}{g(x)\ln(a)}\, g'(x)$
$\dfrac{d}{dx}\left[\cot^{-1}(x)\right] = \dfrac{-1}{1+x^2}$	$\dfrac{d}{dx}\left[\cot^{-1}\left(g(x)\right)\right] = \dfrac{-1}{1+(g(x))^2}\, g'(x)$
$\dfrac{d}{dx}\left[\csc^{-1}(x)\right] = \dfrac{-1}{\lvert x\rvert\sqrt{x^2-1}}$	$\dfrac{d}{dx}\left[\csc^{-1}\left(g(x)\right)\right] = \dfrac{-1}{\lvert g(x)\rvert\sqrt{(g(x))^2-1}}\, g'(x)$

Not used extensively on test

Other Concepts

Be sure you have a command of the following topics.

- Use the limit definition to find the derivative of a function. .... (Section 3.1)
- Given the graph of $f(x)$, sketch the graph of $f'(x)$. .... (Section 3.2)
- Apply derivative rules to find derivatives of various functions. .... (Sections 3.3, 3.4. 3.5, 3.7, 3.9, 3.10)
- Compute higher derivatives. (Second derivative, third derivative, etc.) .... (Section 3.3)
- Use derivatives to find slopes of tangent lines or equations of tangent lines. .... (Sections 3.3–3.10)
- Solve velocity problems. .... (Section 3.6)

  If an object moving on a straight line has position $s(t)$ at time t, then

  - Position at time t is $s(t)$.
  - Velocity at time t is $v(t) = s'(t)$.
  - Acceleration at time t is $a(t) = v'(t) = s''(t)$.

  Velocity is positive when position $s(t)$ increases and negative when $s(t)$ decreases.
  The *speed* of the object at time t is $|v(t)|$, i.e., speed is the absolute value of velocity.

- Implicit differentiation. .... (Section 3.8)
- Logarithmic differentiation. .... (Section 3.9)
- Solve related rates problems. .... (Section 3.11)
- Use and apply properties of exponents and logarithms.
- Work competently with trig functions.
- Use algebra effectively.

Important Note

This review sheet stresses *facts*. You will need to be able to *apply* the facts to solve problems.