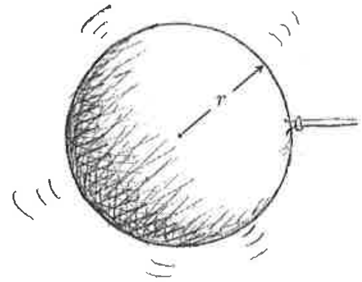
Name: Richard

MATH 200 – QUIZ 8 ⚀

I'm in the Thurs11 Thurs12 Thurs1 or Fri10 recitation. (Circle one)

October 24, 2012

1. A spherical balloon is inflated at a rate of 100π cubic feet per minute.



(a) How fast is the radius increasing at the instant that the radius is 5 feet?

Know $\frac{dV}{dt} = 100\pi$

Want $\frac{dr}{dt}$ when $r=5$

$V = \frac{4}{3}\pi r^3$

$$\frac{d}{dt}[V] = \frac{d}{dt}\left[\frac{4}{3}\pi r^3\right]$$

$$\frac{dV}{dt} = 4\pi r^2 \frac{dr}{dt}$$

$$100\pi = 4\pi (5)^2 \frac{dr}{dt}$$

$$\frac{dr}{dt} = \frac{100\pi}{100\pi} = \boxed{1 \text{ ft/min}}$$

(b) How fast is the surface area increasing at the instant that the radius is 5 feet?

$S = 4\pi r^2$

$$\frac{d}{dt}[S] = \frac{d}{dt}\left[4\pi r^2\right]$$

from above, $\frac{dr}{dt} = 1$

$$\frac{dS}{dt} = 8\pi r \frac{dr}{dt} = 8\pi \cdot 5 \cdot 1 = \boxed{40\pi \text{ square feet/min}}$$

(For a sphere of radius r is the volume is $V = \frac{4}{3}\pi r^3$, and the surface area is $S = 4\pi r^2$.)

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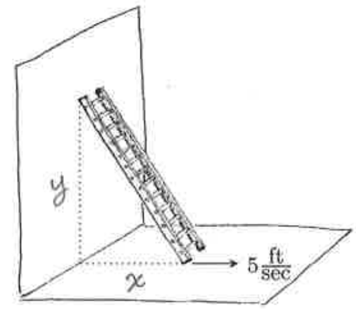
1. A 13-foot ladder is leaning against a wall, as illustrated, when its base begins to slide away from the wall at a rate of 5 feet per second. How fast is the top of the ladder sliding down the wall when the base of the ladder is 12 feet from the wall?



Know $\frac{dx}{dt}$

Want $\frac{dy}{dt}$ when $x = 12$

$x^2 + y^2 = 13^2$

$$\frac{d}{dt}\left[x^2 + y^2\right] = \frac{d}{dt}\left[13^2\right]$$

$$2x\frac{dx}{dt} + 2y\frac{dy}{dt} = 0$$

$$2y\frac{dy}{dt} = -2x\frac{dx}{dt}$$

$$\frac{dy}{dt} = -\frac{x}{y}\frac{dx}{dt}$$

$$= -\frac{12}{y} \cdot 5$$

$$= -\frac{12}{5} \cdot 5$$

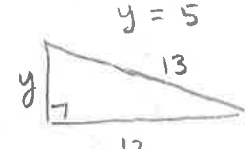
$$= \boxed{-12 \text{ ft/sec}}$$

To find y:

$12^2 + y^2 = 13^2$

$y^2 = 169 - 144 = 25$

$y = 5$



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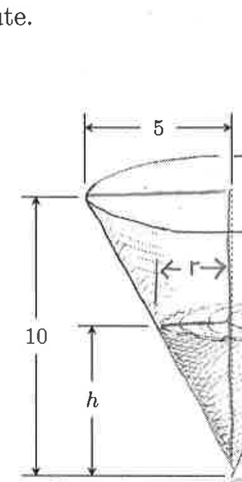
1. Water flows into a conical tank (see illustration) at a rate of 9 cubic feet per minute. How fast is the water level h rising when the water is 6 feet deep?
(The volume of a cone of height h and radius r is $V = \frac{1}{3}\pi r^2 h$.)



Know $\frac{dV}{dt} = 9 \text{ ft}^3/\text{min}$

Want $\frac{dh}{dt}$ when $h = 6$ ft.

$$V = \frac{1}{3}\pi r^2 h$$

$$V = \frac{1}{3}\pi\left(\frac{h}{2}\right)^2 h \quad \text{(see *)}$$

$$V = \frac{\pi}{12}h^3$$

$$\frac{d}{dt}[V] = \frac{d}{dt}\left[\frac{\pi}{12}h^3\right]$$

$$\frac{dV}{dt} = \frac{\pi}{4}h^2\frac{dh}{dt}$$

$$9 = \frac{\pi}{4}\cdot 6^2\frac{dh}{dt}$$

$$9 = 9\pi\frac{dh}{dt}$$

$$\boxed{\frac{dh}{dt} = \frac{9}{9\pi} = \frac{1}{\pi} \text{ ft/min}}$$

By similar △'s:

$$\frac{5}{10} = \frac{r}{h}$$

Thus $\boxed{r = \frac{h}{2}}$ *

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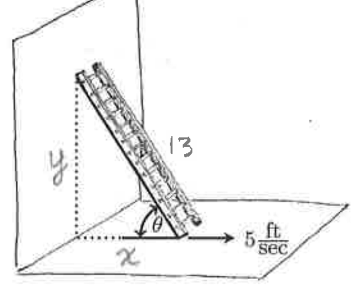
1. A 13-foot ladder is leaning against a wall, as illustrated, when its base begins to slide away from the wall at a rate of 5 feet per second. At what rate is the angle θ changing when the base is 12 feet from the wall?



Know $\frac{dx}{dt} = 5$ ft/sec

Want $\frac{d\theta}{dt}$ when $x = 5$

From picture:

$$\cos(\theta) = \frac{\text{ADJ}}{\text{HYP}} = \frac{x}{13}$$

$$\cos(\theta) = \frac{x}{13}$$

$$\frac{d}{dt}[\cos(\theta)] = \frac{d}{dt}\left[\frac{x}{13}\right]$$

$$-\sin(\theta)\frac{d\theta}{dt} = \frac{1}{13}\frac{dx}{dt}$$

$$\frac{d\theta}{dt} = \frac{-1}{13\sin(\theta)}\frac{dx}{dt}$$

$$= \frac{-5}{13\sin(\theta)}$$

$$= \frac{-5}{13\cdot\frac{5}{13}}$$

$$= \boxed{-1 \text{ radians/sec}}$$

Finding $\sin(\theta)$

$$\sin(\theta) = \frac{\text{OPP}}{\text{HYP}} = \frac{5}{13}$$