

1. This problem involves the equation $x^2 + xy + y^2 = 7$.

(a) Use implicit differentiation to find y' .

$$\frac{d}{dx}[x^2 + xy + y^2] = \frac{d}{dx}[7]$$

$$2x + 1 \cdot y + xy' + 2yy' = 0$$

$$xy' + 2yy' = -2x - y$$

$$y'(x + 2y) = -2x - y$$

$$y' = \frac{-2x - y}{x + 2y}$$

(b) Use your answer from (a) to find the slope of the tangent to the equation's graph at $(-2, 3)$.

$$y' \Big|_{(x,y)=(-2,3)} = \frac{-2(-2) - 3}{-2 + 2 \cdot 3} = \boxed{\frac{1}{4}}$$

2. Use logarithmic differentiation to find the derivative of $y = (x-1)^x$.

$$\ln(y) = \ln((x-1)^x)$$

$$\ln(y) = x \ln(x-1)$$

$$\frac{d}{dx}[\ln(y)] = \frac{d}{dx}[x \ln(x-1)]$$

$$\frac{y'}{y} = 1 \cdot \ln(x-1) + x \frac{1}{x-1}$$

$$y' = y \left(\ln(x-1) + \frac{x}{x-1} \right) = \boxed{(x-1)^x \left(\ln(x-1) + \frac{x}{x-1} \right)}$$