$\frac{10}{10}$

MATH 200 – Quiz 6 ☽

Name: *Richard* February 26, 2015

1. Suppose $f(x) = (x^2 - \pi^2)\cos(x)$.

   (a) $$f'(x) = \frac{d}{dx}[x^2-\pi^2]\cdot\cos(x) + (x^2-\pi^2)\cdot\frac{d}{dx}[\cos(x)]$$
   $$= (2x-0)\cdot\cos(x) + (x^2-\pi^2)\cdot(-\sin(x))$$
   $$= \boxed{2x\cos(x) - (x^2-\pi^2)\sin(x))}$$

   (b) Find the equation of the tangent line to the graph of $f(x)$ at the point $(\pi, f(\pi))$. $= \big(\pi, (\pi^2-\pi^2)\cos(\pi)\big) = (\pi, 0)$

   The slope of the line is $f'(\pi) = 2\pi\cos(\pi) - (\pi^2-\pi^2)\sin(\pi) = -2\pi - 0\cdot 0 = -2\pi$.

   Using the point-slope formula for the equation of a straight line, we get

   $$\begin{aligned} y - y_0 &= m(x - x_0) \\ y - 0 &= -2\pi(x-\pi) \\ y &= -2\pi x + 2\pi^2 \end{aligned}$$

   Answer: $\boxed{y = -2\pi x + 2\pi^2}$

2. If $z = \dfrac{5}{w} + \dfrac{\tan(w)}{w+1}$, then $\dfrac{dz}{dw} =$ Note: $z = 5w^{-1} + \dfrac{\tan(w)}{w+1}$.

   Thus $\dfrac{dz}{dw} = -5w^{-2} + \dfrac{\sec^2(w)(w+1) - \tan(w)\cdot 1}{(w+1)^2} = \boxed{-\dfrac{5}{w^2} + \dfrac{(w+1)\sec^2(w) - \tan(w)}{(w+1)^2}}$

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MATH 200 – Quiz 6 ☾

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1. Suppose $f(x) = \dfrac{\sin(x)}{x}$.

   (a) $$f'(x) = \frac{\frac{d}{dx}[\sin(x)]\cdot x - \sin(x)\cdot\frac{d}{dx}[x]}{x^2} = \frac{\cos(x)\cdot x - \sin(x)\cdot 1}{x^2} = \frac{x\cos(x) - \sin(x)}{x^2}$$

   (b) Find the equation of the tangent line to the graph of $f(x)$ at the point $(\pi, f(\pi))$. $= \left(\pi, \frac{\sin(\pi)}{\pi}\right) = (\pi, 0)$

   The slope of the line is $f'(\pi) = \dfrac{\pi\cos(\pi) - \sin(\pi)}{\pi^2} = \dfrac{\pi(-1)-0}{\pi^2} = \dfrac{-\pi}{\pi^2} = -\dfrac{1}{\pi}$.

   Using the point-slope formula for the equation of a straight line, we get

   $$\begin{aligned} y - y_0 &= m(x - x_0) \\ y - 0 &= -\frac{1}{\pi}(x-\pi) \\ y &= -\frac{1}{\pi}x + 1 \end{aligned}$$

   Answer: $\boxed{y = -\dfrac{1}{\pi}x + 1}$

2. If $z = \sqrt{w} + 5(w+1)\sec(w)$, then $\dfrac{dz}{dw} =$ Note: $z = w^{1/2} + 5(w+1)\sec(w)$.

   Thus $\dfrac{dz}{dw} = \dfrac{1}{2}w^{-1/2} + 5\sec(w) + 5(w+1)\sec(w)\tan(w) = \boxed{\dfrac{1}{2\sqrt{w}} + 5\sec(w) + 5(w+1)\sec(w)\tan(w)}$