Name: Richard

MATH 200 – Quiz 5

February 14, 2013

1. Suppose $f(x) = \frac{1}{x^2}$. Use the limit definition of the derivative to find $f'(x)$. Please show all work.

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h} = \lim_{h \to 0} \frac{\frac{1}{(x+h)^2} - \frac{1}{x^2}}{h}$$

$$= \lim_{h \to 0} \frac{\frac{1}{(x+h)^2} - \frac{1}{x^2}}{h} \frac{(x+h)^2 x^2}{(x+h)^2 x^2} = \lim_{h \to 0} \frac{x^2 - (x+h)^2}{h(x+h)^2 x^2}$$

$$= \lim_{h \to 0} \frac{x^2 - (x^2 + 2xh + h^2)}{h(x+h)^2 x^2} = \lim_{h \to 0} \frac{x^2 - x^2 - 2xh - h^2}{h(x+h)^2 x^2}$$

$$= \lim_{h \to 0} \frac{-2xh - h^2}{h(x+h)^2 x^2} = \lim_{h \to 0} \frac{\cancel{h}(-2x - h)}{\cancel{h}(x+h)^2 x^2} = \lim_{h \to 0} \frac{-2x - h}{(x+h)^2 x^2}$$

$$= \frac{-2x - 0}{(x+0)^2 x^2} = \frac{-2x}{x^2 x^2} = \frac{-2x}{x^4} = \boxed{-\frac{2}{x^3}} \quad \text{Thus } \boxed{f'(x) = -\frac{2}{x^3}}$$

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1. Suppose $f(x) = 4 - 3x^2$. Use the limit definition of the derivative to find $f'(x)$. Please show all work.

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h} = \lim_{h \to 0} \frac{4 - 3(x+h)^2 - (4 - 3x^2)}{h}$$

$$= \lim_{h \to 0} \frac{4 - 3(x^2 + 2xh - h^2) - (4 - 3x^2)}{h}$$

$$= \lim_{h \to 0} \frac{4 - 3x^2 - 6xh - 3h^2 - 4 + 3x^2}{h}$$

$$= \lim_{h \to 0} \frac{-6xh - 3h^2}{h} = \lim_{h \to 0} \frac{h(-6x - 3h)}{h}$$

$$= \lim_{h \to 0} (-6x - 3h) = -6x - 3 \cdot 0 = \boxed{-6x}$$

Therefore $\boxed{f'(x) = -6x}$

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MATH 200 – QUIZ 5

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1. Suppose $f(x) = \sqrt{6x}$. Use the limit definition of the derivative to find $f'(x)$. Please show all work.

$$f'(x) = \lim_{h\to 0} \frac{f(x+h) - f(x)}{h} = \lim_{h\to 0} \frac{\sqrt{6(x+h)} - \sqrt{6x}}{h}$$

$$= \lim_{h\to 0} \frac{\sqrt{6(x+h)} - \sqrt{6x}}{h} \; \frac{\sqrt{6(x+h)} + \sqrt{6x}}{\sqrt{6(x+h)} + \sqrt{6x}}$$

$$= \lim_{h\to 0} \frac{6(x+h) - 6x}{h\left(\sqrt{6(x+h)} + \sqrt{6x}\right)} = \lim_{h\to 0} \frac{6x + 6h - 6x}{h\left(\sqrt{6x+6h} - \sqrt{6x}\right)}$$

$$= \lim_{h\to 0} \frac{6h}{h\left(\sqrt{6x+6h} - \sqrt{6x}\right)} = \lim_{h\to 0} \frac{6}{\sqrt{6x+6h} - \sqrt{6x}}$$

$$= \frac{6}{\sqrt{6x+0} + \sqrt{6x}} = \frac{6}{2\sqrt{6x}} = \boxed{\frac{3}{\sqrt{6x}}}$$ Thus $$\boxed{f'(x) = \frac{3}{\sqrt{6x}}}$$

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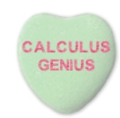

1. Suppose $f(x) = \frac{1}{7x}$. Use the limit definition of the derivative to find $f'(x)$. Please show all work.

$$f'(x) = \lim_{h\to 0} \frac{f(x+h) - f(x)}{h} = \lim_{h\to 0} \frac{\frac{1}{7(x+h)} - \frac{1}{7x}}{h}$$

$$= \lim_{h\to 0} \frac{\frac{1}{7(x+h)} - \frac{1}{7x}}{h} \; \frac{7x(x+h)}{7x(x+h)}$$

$$= \lim_{h\to 0} \frac{x - (x+h)}{h\,7x\,(x+h)} = \lim_{h\to 0} \frac{-h}{h\,7x(x+h)}$$

$$= \lim_{h\to 0} \frac{-1}{7x(x+h)} = \frac{-1}{7x(x+0)} = \boxed{\frac{-1}{7x^2}}$$

Therefore $$\boxed{f'(x) = \frac{-1}{7x^2}}$$