Name: Richard

QUIZ 5

MATH 200
September 7, 2023

Directions: Show all steps (within reason). Simplify your answer.

1. This problem concerns the function $f(x) = \dfrac{3}{x-2}$. Do **either** (a) or (b) below. (Your choice.)

   (a) Use a limit definition of the derivative to find $f'(x)$. Then use your answer to find $f'(4)$.

   (b) Use a limit definition of the derivative at a point to find $f'(4)$.

(a) $$f'(x) = \lim_{z\to x} \frac{f(z)-f(x)}{z-x} = \lim_{z\to x} \frac{\frac{3}{z-2} - \frac{3}{x-2}}{z-x}$$

$$= \lim_{z\to x} \frac{\frac{3}{z-2} - \frac{3}{x-2}}{z-x} \cdot \frac{(z-2)(x-2)}{(z-2)(x-2)}$$

$$= \lim_{z\to x} \frac{3(x-2)-3(z-2)}{(z-x)(z-2)(x-2)} = \lim_{z\to x} \frac{3x-6-3z+6}{(z-x)(z-2)(x-2)}$$

$$= \lim_{z\to x} \frac{3x-3z}{(z-x)(z-2)(x-2)} = \lim_{z\to x} \frac{-3\cancel{(z-x)}}{\cancel{(z-x)}(z-2)(x-2)}$$

$$= \lim_{z\to x} \frac{-3}{(z-2)(x-2)} = \frac{-3}{(x-2)(x-2)} = \frac{-3}{(x-2)^2}$$

Thus $\boxed{f'(x) = \dfrac{-3}{(x-2)^2}}$ so $f'(4) = \dfrac{-3}{(4-2)^2} = \dfrac{-3}{2^2} = \boxed{-\dfrac{3}{4}}$

(b) $$f'(4) = \lim_{z\to 4} \frac{f(z)-f(4)}{z-4} = \lim_{z\to 4} \frac{\frac{3}{z-2} - \frac{3}{4-2}}{z-4}$$

$$= \lim_{z\to 4} \frac{\frac{3}{z-2} - \frac{3}{2}}{z-4} = \lim_{z\to 4} \frac{\frac{3}{z-2} - \frac{3}{2}}{z-4} \cdot \frac{2(z-2)}{2(z-2)}$$

$$= \lim_{z\to 4} \frac{6-3(z-2)}{(z-4)2(z-2)} = \lim_{z\to 4} \frac{6-3z+6}{2(z-4)(z-2)}$$

$$= \lim_{z\to 4} \frac{12-3z}{2(z-4)(z-2)} = \lim_{z\to 4} \frac{-3\cancel{(z-4)}}{2\cancel{(z-4)}(z-2)} = \lim_{z\to 4} \frac{-3}{2(z-2)} = \boxed{\frac{-3}{4}}$$

Name: Richard

QUIZ 5

MATH 200
September 7, 2023

Directions: Show all steps (within reason). Simplify your answer.

1. This problem concerns the function $f(x) = \sqrt{x+9}$. Do **either** (a) or (b) below. (Your choice.)
   (a) Use a limit definition of the derivative to find $f'(x)$. Then use your answer to find $f'(16)$.
   (b) Use a limit definition of the derivative at a point to find $f'(16)$.

(a) $$f'(x) = \lim_{z \to x} \frac{f(z)-f(x)}{z-x} = \lim_{z\to x} \frac{\sqrt{z+9}-\sqrt{x+9}}{z-x}$$

$$= \lim_{z\to x} \frac{\sqrt{z+9}-\sqrt{x+9}}{z-x} \cdot \frac{\sqrt{z+9}+\sqrt{x+9}}{\sqrt{z+9}+\sqrt{x+9}}$$

$$= \lim_{z\to x} \frac{(z+9)-(x+9)}{(z-x)(\sqrt{z+9}+\sqrt{x+9})} = \lim_{z\to x} \frac{z+9-x-9}{(z-x)(\sqrt{z+9}+\sqrt{x+9})}$$

$$= \lim_{z\to x} \frac{\cancel{z-x}}{\cancel{(z-x)}(\sqrt{z+9}+\sqrt{x+9})} = \lim_{z\to x} \frac{1}{\sqrt{z+9}+\sqrt{x+9}}$$

$$= \frac{1}{\sqrt{x+9}+\sqrt{x+9}} = \frac{1}{2\sqrt{x+9}}$$

Thus $\boxed{f'(x) = \frac{1}{2\sqrt{x+9}}}$ so $f'(16) = \frac{1}{2\sqrt{16+9}} = \frac{1}{2\sqrt{25}} = \frac{1}{2\cdot 5} = \boxed{\frac{1}{10}}$

(b) $$f'(16) = \lim_{z\to 16} \frac{f(z)-f(16)}{z-16} = \lim_{z\to 16} \frac{\sqrt{z+9}-\sqrt{16+9}}{z-16}$$

$$= \lim_{z\to 16} \frac{\sqrt{z+9}-\sqrt{25}}{z-16} = \lim_{z\to 16} \frac{\sqrt{z+9}-5}{z-16}$$

$$= \lim_{z\to 16} \frac{\sqrt{z+9}-5}{z-16} \cdot \frac{\sqrt{z+9}+5}{\sqrt{z+9}+5}$$

$$= \lim_{z\to 16} \frac{z+9-25}{(z-16)(\sqrt{z+9}+5)} = \lim_{z\to 16} \frac{\cancel{z-16}}{\cancel{(z-16)}(\sqrt{z+9}+5)}$$

$$= \lim_{z\to 16} \frac{1}{\sqrt{z+9}+5} = \frac{1}{\sqrt{16+9}+5} = \frac{1}{\sqrt{25}+5} = \frac{1}{5+5} = \boxed{\frac{1}{10}}$$