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QUIZ 4

MATH 200, SECTION 3
June 16, 2021

1. (10 points) Suppose $f(x)$ is a function for which $f'(x) = 3x^2 + 4$ and $f(2) = 7$. Find $f(x)$.

$$f(x) = \int (3x^2 + 4x)\,dx = 3\frac{x^3}{3} + 4x + C = x^3 + 4x + C$$

i.e. $f(x) = x^3 + 4x + C$. To find C, plug in $x=2$:

$$7 = f(2) = 2^3 + 4\cdot 2 + C = 16 + C, \text{ so } C = 7 - 16 = -9$$

Thus $\boxed{f(x) = x^3 + 4x - 9}$

2. (10 points) Suppose f and g are functions for which $\int_0^5 f(x)\,dx = 3$, $\int_5^7 f(x)\,dx = -2$, and $\int_0^7 g(x)\,dx = 6$.

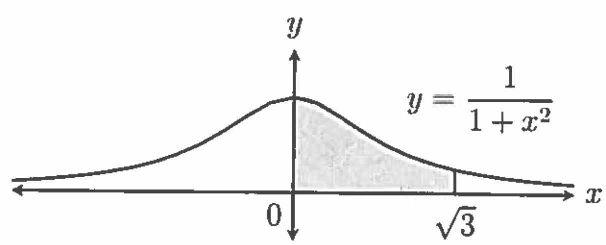
Find $\int_0^7 (f(x) - 3g(x))\,dx$

Note: $\int_0^7 f(x)\,dx = \int_0^5 f(x)\,dx + \int_5^7 f(x)\,dx = 3 - 2 = 1$

Now, $\int_0^7 (f(x) - 3g(x))\,dx = \int_0^7 f(x)\,dx - 3\int_0^7 g(x)\,dx$

$$= 1 - 3\cdot 6 = \boxed{-17}$$

3. (6 points) Find the indicated (shaded) area below the graph of $y = \dfrac{1}{1+x^2}$.



$$A = \int_0^{\sqrt{3}} \frac{1}{1+x^2}\,dx$$

$$= \left[\tan^{-1}(x)\right]_0^{\sqrt{3}}$$

$$= \tan^{-1}(\sqrt{3}) - \tan^{-1}(0) = \frac{\pi}{3} - 0 = \boxed{\frac{\pi}{3}}$$

Please see other side.

4. (24 points) Use the fundamental theorem of calculus to find the following definite integrals.

(a) $\int_{-2}^{2} (x^3 - x)\, dx = \left[\frac{x^4}{4} - \frac{x^2}{2}\right]_{-2}^{2} = \left(\frac{2^4}{4} - \frac{2^2}{2}\right) - \left(\frac{(-2)^4}{4} - \frac{(-2)^2}{2}\right)$

$= \left(\frac{16}{4} - \frac{4}{2}\right) - \left(\frac{16}{4} - \frac{4}{2}\right) = \boxed{0}$

(b) $\int_{1}^{e} \frac{2}{x}\, dx = 2\int_1^e \frac{1}{x}\, dx = 2\left[\ln|x|\right]_1^e$

$= 2\ln|e| - 2\ln|1|$

$= 2\cdot 1 - 2\cdot 0 = \boxed{2}$

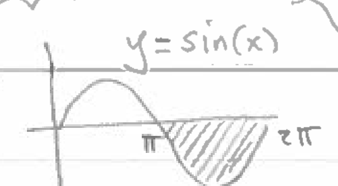
(c) $\int_{0}^{1} (1+\sqrt{x})\, dx = \int_0^1 (1 + x^{\frac{1}{2}})\, dx = \left[x + \frac{1}{\frac{1}{2}+1} x^{\frac{1}{2}+1}\right]_0^1$

$= \left[x + \frac{1}{\frac{3}{2}} x^{3/2}\right]_0^1 = \left[x + \frac{2}{3}\sqrt{x}^3\right]_0^1 = \left(1 + \frac{2}{3}\sqrt{1}^3\right) - \left(0 + \frac{2}{3}\sqrt{0}^3\right)$

$= 1 + \frac{2}{3} - 0 = \boxed{\frac{5}{3}}$

(d) $\int_{\pi}^{2\pi} \sin(x)\, dx = \left[-\cos(x)\right]_{\pi}^{2\pi} = -\cos(2\pi) - (-\cos(\pi))$

$= -1 - (-(-1)) = \boxed{-2}$



Expect negative answer!