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QUIZ 3

MATH 200, SECTION 3
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1. (14 points) An object moving on a straight line is $s(t) = t^3 - 3t^2$ feet from its starting point at time t. Find the object's acceleration at the instant its velocity is -3 feet per second.

Velocity at time t is $v(t) = s'(t) = 3t^2 - 6t$.
To find when velocity is -3 ft/sec, solve

$$\begin{aligned} v(t) &= -3 \\ 3t^2 - 6t &= -3 \\ 3t^2 - 6t + 3 &= 0 \\ 3(t^2 - 2t + 1) &= 0 \\ 3(t-1)(t-1) &= 0 \quad \Rightarrow t = 1 \end{aligned}$$

Thus velocity is -3 ft/sec when $t = 1$ sec.
Now, acceleration is $a(t) = v'(t) = 6t - 6$
So at time $t = 1$, acceleration is $a(1) = 6 \cdot 1 - 6 = \boxed{0 \text{ ft/sec}^2}$

2. (18 points) This problem concerns the equation $x^2 = y\cos(y)$.

(a) Use implicit differentiation to find y'.

$y = f(x)$

$$\begin{aligned} \frac{d}{dx}\left[x^2\right] &= \frac{d}{dx}\left[y\cos(y)\right] \\ 2x &= y' \cdot \cos(y) + y(-\sin(y)\,y') \\ 2x &= y'(\cos(y) - y\sin(y)) \end{aligned}$$

$$\boxed{y' = \frac{2x}{\cos(y) - y\sin(y)}}$$

(b) Use part (a) to find the slope of the tangent to the graph of $x^2 = y\cos(y)$ at the point $(\sqrt{\pi}, -\pi)$.

$$y'\Big|_{(x,y)=(\sqrt{\pi},\pi)} = \frac{2\sqrt{\pi}}{\cos(-\pi) - (-\pi\sin(-\pi))} = \frac{2\sqrt{\pi}}{-1-0} = \boxed{-2\sqrt{\pi}}$$

Please see other side.

3. (18 points) This problem concerns the function $f(x) = x^3 e^x$

(a) Find the critical points of f.

$$f'(x) = 3x^2e^x + x^3e^x = 0$$

$$e^x(3x^2 + x^3) = 0$$

$$e^x x^2(3+x) = 0$$

$x = 0$ $\qquad$ $x = -3$

$\boxed{\text{Critical points are } x=0,\ x=-3}$

(b) State the interval(s) on which f increases.

-3 $\qquad$ 0

$- - - - \mid + + + + + + \mid + + +$ $\quad f'(x) = e^x x^2(3+x)$

$\boxed{f(x) \text{ increases on } (-3, 0) \cup (0, \infty)}$

(c) State the interval(s) on which f decreases.

$\boxed{f(x) \text{ decreases on } (-\infty, -3)}$

(d) State the locations (x coordinates) of any local minima of f.

By 1st derivative test, $\boxed{\text{local min at } x = -3}$

(e) State the locations (x coordinates) of any local maxima of f.

By 1st derivative test, $\boxed{\text{no local max}}$

(f) Identify the locations of any global extrema of $f(x)$ on the *open* interval $(-8, -1)$.

The only critical point in this interval is $x = -3$ and by (d) above f has a local miniminum at $x = -3$. Since there is only one critical point in the interval, this is a global minimum at $x = -3$. There is no global maximum