

1. (14 points) An object moving on a straight line is $s(t) = t^3 - 3t^2$ feet from its starting point at time t . Find the object's acceleration at the instant its velocity is -3 feet per second.

Velocity at time t is $V(t) = s'(t) = 3t^2 - 6t$.

To find when velocity is -3 ft/sec, solve

$$\begin{aligned} V(t) &= -3 \\ 3t^2 - 6t &= -3 \\ 3t^2 - 6t + 3 &= 0 \\ 3(t^2 - 2t + 1) &= 0 \\ 3(t-1)(t-1) &= 0 \Rightarrow t=1 \end{aligned}$$

Thus velocity is -3 ft/sec when $t = 1$ sec.

Now, acceleration is $a(t) = V'(t) = 6t - 6$

So at time $t = 1$, acceleration is $a(1) = 6 \cdot 1 - 6 = 0$ ft/sec²

2. (18 points) This problem concerns the equation $x^2 = y \cos(y)$.

(a) Use implicit differentiation to find y' .

$$y = f(x)$$

$$\frac{d}{dx} [x^2] = \frac{d}{dx} [y \cos(y)]$$

$$2x = y' \cos(y) + y(-\sin(y) y')$$

$$2x = y'(\cos(y) - y \sin(y))$$

$$y' = \frac{2x}{\cos(y) - y \sin(y)}$$

- (b) Use part (a) to find the slope of the tangent to the graph of $x^2 = y \cos(y)$ at the point $(\sqrt{\pi}, -\pi)$.

$$y' \Big|_{(x,y) = (\sqrt{\pi}, -\pi)} = \frac{2\sqrt{\pi}}{\cos(-\pi) - (-\pi \sin(-\pi))} = \frac{2\sqrt{\pi}}{-1 - 0} = -2\sqrt{\pi}$$

3. (18 points) This problem concerns the function $f(x) = x^3 e^x$

(a) Find the critical points of f .

$$f'(x) = 3x^2 e^x + x^3 e^x = 0$$

$$e^x (3x^2 + x^3) = 0$$

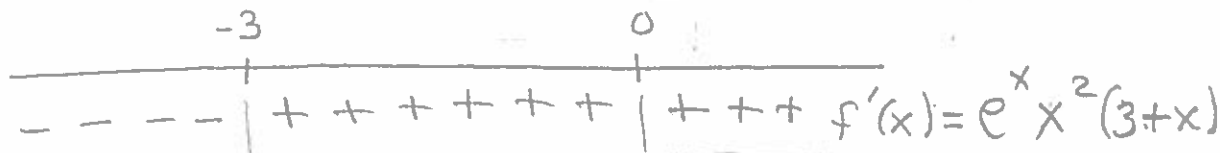
$$e^x x^2 (3+x) = 0$$

$$\downarrow \\ x=0$$

$$\downarrow \\ x=-3$$

Critical points
are $x=0$, $x=-3$

(b) State the interval(s) on which f increases.



$f(x)$ increases on $(-3, 0) \cup (0, \infty)$

(c) State the interval(s) on which f decreases.

$f(x)$ decreases on $(-\infty, -3)$

(d) State the locations (x coordinates) of any local minima of f .

By 1st derivative test, local min at $x = -3$

(e) State the locations (x coordinates) of any local maxima of f .

By 1st derivative test, no local max

(f) Identify the locations of any global extrema of $f(x)$ on the open interval $(-8, -1)$.

The only critical point in this interval is $x = -3$ and by (d) above f has a local minimum at $x = -3$. Since there is only one critical point in the interval, this is a global minimum at $x = -3$. There is no global maximum