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QUIZ 2

MATH 200, SECTION 3
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1. (15 points) Use the **limit definition** of the derivative to find the derivative of $f(x) = \frac{x}{x+1}$.

$$f'(x) = \lim_{z\to x} \frac{f(z)-f(x)}{z-x} = \lim_{z\to x} \frac{\frac{z}{z+1} - \frac{x}{x+1}}{z-x}$$

$$= \lim_{z\to x} \frac{\frac{z}{z+1} - \frac{x}{x+1}}{z-x} \cdot \frac{(z+1)(x+1)}{(z+1)(x+1)}$$

$$= \lim_{z\to x} \frac{z(x+1)-x(z+1)}{(z-x)(z+1)(x+1)} = \lim_{z\to x} \frac{zx+z-xz-x}{(z-x)(z+1)(x+1)}$$

$$= \lim_{z\to x} \frac{\cancel{z-x}}{(\cancel{z-x})(z+1)(x+1)} = \lim_{z\to x} \frac{1}{(z+1)(x+1)} = \boxed{\frac{1}{(x+1)^2}}$$

2. (35 points) Use derivative rules to find derivatives of the following functions.

(a) $f(x) = \frac{x}{x+1}$

$$f'(x) = \frac{1\cdot(x+1) - x(1+0)}{(x+1)^2} = \frac{x+1-x}{(x+1)^2} = \boxed{\frac{1}{(x+1)^2}}$$

(b) $f(x) = 5x^6 + e^x + 3$

$$f'(x) = \boxed{30x^5 + e^x}$$

(c) $f(x) = \sqrt{x} + \tan(x) = x^{\frac{1}{2}} + \tan(x)$

$$f'(x) = \frac{1}{2}x^{\frac{1}{2}-1} + \sec^2(x)$$

$$= \frac{1}{2}x^{-1/2} + \sec^2(x) = \boxed{\frac{1}{2\sqrt{x}} + \sec^2(x)}$$

(d) $f(x) = \pi x\cos(x)$

$$f'(x) = \pi\cos(x) + \pi x(-\sin(x)) = \boxed{\pi\cos(x) - \pi x\sin(x)}$$

(e) $f(x) = \frac{xe^x}{x^2+1}$

$$f'(x) = \frac{\frac{d}{dx}[xe^x](x^2+1) - xe^x(2x+0)}{(x^2+1)^2}$$

$$= \boxed{\frac{(e^x+xe^x)(x^2+1) - 2x^2e^x}{(x^2+1)^2}} = \boxed{\frac{e^x((1+x)(x^2+1)-2x^2)}{(x^2+1)^2}} = \boxed{\frac{e^x(x^3-x^2+x+1)}{(x^2+1)^2}}$$