Name: ______________________________ **QUIZ 25** **MATH 200**

April 28, 2022

1. $$\int (1+\sin(x))^2\cos(x)\,dx = \int u^2du = \frac{u^3}{3}+C = \boxed{\frac{(1+\sin(x))^3}{3}+C}$$

$$\boxed{\begin{array}{l} u = 1+\sin(x) \\ \dfrac{du}{dx} = \cos(x) \\ du = \cos(x)dx \end{array}}$$

2. $$\int \frac{\sin(2x)}{\cos^5(2x)}\,dx = \int \frac{1}{\big(\cos(2x)\big)^5}\sin(2x)\,dx = \int \big(\cos(2x)\big)^{-5}\sin(2x)\,dx = \int u^{-5}\left(-\frac{1}{2}\right)du = -\frac{1}{2}\int u^{-5}du$$

$$\boxed{\begin{array}{l} u = \cos(2x) \\ \dfrac{du}{dx} = -\sin(2x)\cdot 2 \\ du = -2\sin(2x)dx \\ -\dfrac{1}{2}du = \sin(2x)dx \end{array}}$$

$$= -\frac{1}{2}\cdot\frac{u^{-4}}{-4}+C = \frac{1}{8u^4}+C = \boxed{\frac{1}{8\cos^4(2x)}+C}$$

3. $$\int_0^1 \left(1+x^2\right)^3 2x\,dx = \int_{1+0^2}^{1+1^2} u^3\,du = \int_1^2 u^3\,du = \left[\frac{u^4}{4}\right]_1^2 = \frac{2^4}{4}-\frac{1^4}{4} = \frac{16}{4}-\frac{1}{4} = \boxed{\frac{15}{4}}$$

$$\boxed{\begin{array}{l} u = 1+x^2 \\ \dfrac{du}{dx} = 2x \\ du = 2x\,dx \end{array}}$$

4. Find the area under the graph of $\sec^2(2x)$ between $x{=}0$ and $x{=}\pi/8$.

$$\text{Area} = \int_0^{\pi/8} \sec^2(2x)\,dx = \int_{2\cdot 0}^{2\cdot\pi/8} \sec^2(u)\frac{1}{2}\,du = \frac{1}{2}\int_0^{\pi/4}\sec^2(u)\,du = \frac{1}{2}\Big[\tan(u)\Big]_0^{\pi/4} =$$

$$\boxed{\begin{array}{l} u = 2x \\ \dfrac{du}{dx} = 2 \\ du = 2\,dx \\ \dfrac{1}{2}\,du = dx \end{array}}$$

$$= \frac{1}{2}\Big[\tan(u)\Big]_0^{\pi/4} = \frac{1}{2}\left(\tan(\pi/4)-\tan(0)\right) = \frac{1}{2}(1-0) = \boxed{\frac{1}{2}}$$

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1. $$\int e^{x^2+x}(2x+1)\,dx = \int e^u du = e^u + C = \boxed{e^{x^2+x}+C}$$

$$\boxed{\begin{aligned} &u = x^2 + x \\ &\frac{du}{dx} = 2x+1 \\ &du = (2x+1)\,dx \end{aligned}}$$

2. $$\int \frac{\cos(\sqrt{x})}{\sqrt{x}}\,dx = \int \cos(\sqrt{x})\frac{1}{\sqrt{x}}\,dx = \int \cos(u)2\,du = 2\int \cos(u)\,du = 2\sin(u) + C = \boxed{2\sin(\sqrt{x})+C}$$

$$\boxed{\begin{aligned} &u = \sqrt{x} \\ &\frac{du}{dx} = \frac{1}{2\sqrt{x}} \\ &du = \frac{1}{2\sqrt{x}}dx \\ &2\,du = \frac{1}{\sqrt{x}}dx \end{aligned}}$$

3. $$\int_{-1}^{0} \sqrt{1+x}\,dx = \int_{-1}^{0}(1+x)^{1/2}\,dx = \int_{1+(-1)}^{1+0} u^{1/2}\,du = \int_0^1 u^{1/2}\,du = \left[\frac{u^{1/2+1}}{1/2+1}\right]_0^1 = \left[\frac{u^{3/2}}{3/2}\right]_0^1$$

$$\boxed{\begin{aligned} &u = 1 + x \\ &\frac{du}{dx} = 0+1 \\ &du = dx \end{aligned}}$$

$$= \left[\frac{2\sqrt{u}^3}{3}\right]_0^1 = \frac{2\sqrt{1}^3}{3} - \frac{2\sqrt{0}^3}{3} = \boxed{\frac{2}{3}}$$

4. Find the area under the graph of $\sec^2(2x)$ between $x{=}0$ and $x{=}\pi/8$.

$$\text{Area} = \int_0^{\pi/8} \sec^2(2x)\,dx = \int_{2\cdot 0}^{2\cdot\pi/8} \sec^2(u)\frac{1}{2}\,du = \frac{1}{2}\int_0^{\pi/4}\sec^2(u)\,du = \frac{1}{2}\Big[\tan(u)\Big]_0^{\pi/4} =$$

$$\boxed{\begin{aligned} &u = 2x \\ &\frac{du}{dx} = 2 \\ &du = 2\,dx \\ &\frac{1}{2}\,du = dx \end{aligned}}$$

$$= \frac{1}{2}\Big[\tan(u)\Big]_0^{\pi/4} = \frac{1}{2}\left(\tan(\pi/4) - \tan(0)\right) = \frac{1}{2}(1-0) = \boxed{\frac{1}{2}}$$