Name: Richard

QUIZ 25 ♡

MATH 200
December 13, 2021

1. $\int \sin\left(x^2+x\right)(2x+1)\,dx = \int \sin(u)\,du = -\cos(u) + C = \boxed{-\cos(x^2+x)+C}$

$u = x^2+x$

$\frac{du}{dx} = 2x+1 \implies du = (2x+1)\,dx$

2. $\int \frac{e^{\sqrt{x}}}{\sqrt{x}}\,dx = \int e^{\sqrt{x}} \frac{1}{\sqrt{x}}\,dx = \int e^u \, 2\,du = 2\int e^u\,du$

$= 2e^u + C = \boxed{2e^{\sqrt{x}}+C}$

$u = \sqrt{x} = x^{\frac{1}{2}}$

$\frac{du}{dx} = \frac{1}{2}x^{-1/2} = \frac{1}{2\sqrt{x}}$

$du = \frac{1}{2\sqrt{x}}\,dx \implies 2\,du = \frac{1}{\sqrt{x}}\,dx$

3. $\int_0^{\pi/2} \frac{\cos(x)}{\sin(x)+1}\,dx = \int_0^{\pi/2} \frac{1}{\sin(x)+1}\cos(x)\,dx = \int_{\sin(0)+1}^{\sin(\pi/2)+1} \frac{1}{u}\,du$

$u = \sin(x)+1$

$\frac{du}{dx} = \cos(x)$

$du = \cos(x)\,dx$

$= \int_{0+1}^{1+1} \frac{1}{u}\,du = \int_1^2 \frac{1}{u}\,du$

$= \left[\ln|u|\right]_1^2 = \ln|2| - \ln|1| = \boxed{\ln(2)}$

4. $\int_0^1 \left(x^2+1\right)^3 x\,dx = \int_{0^2+1}^{1^2+1} u^3 \frac{1}{2}\,du$

$u = x^2+1$

$\frac{du}{dx} = 2x$

$du = 2x\,dx$

$\frac{1}{2}du = x\,dx$

$= \frac{1}{2}\int_1^2 u^3\,du = \frac{1}{2}\left[\frac{u^4}{4}\right]_1^2$

$= \frac{1}{2}\left(\frac{2^4}{4} - \frac{1^4}{4}\right) = \frac{1}{2}\left(\frac{16}{4} - \frac{1}{4}\right) = \boxed{\frac{15}{8}}$

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1. $\int e^{x^2+x}(2x+1)\,dx = \int e^u\,du = e^u + C = \boxed{e^{x^2+x} + C}$

$u = x^2 + x$

$\frac{du}{dx} = 2x+1 \Rightarrow du = (2x+1)\,dx$

2. $\int \frac{\ln(2x+1)}{2x+1}\,dx = \int \ln(2x+1)\frac{1}{2x+1}\,dx = \int u\,\frac{1}{2}\,du = \frac{1}{2}\int u^1\,du$

$u = \ln(2x+1)$

$\frac{du}{dx} = \frac{2}{2x+1}$

$du = \frac{2}{2x+1}\,dx \Rightarrow \frac{1}{2}\,du = \frac{1}{2x+1}\,dx$

$= \frac{1}{2}\,\frac{1}{1+1}u^{1+1} + C = \frac{1}{4}u^2 + C$

$= \boxed{\frac{1}{4}\left(\ln(2x+1)\right)^2 + C}$

3. $\int_0^{\sqrt{\pi/4}} \sec^2(x^2)\,x\,dx = \int_{0^2}^{\sqrt{\pi/4}^{\,2}} \sec^2(u)\,\frac{1}{2}\,du = \frac{1}{2}\int_0^{\pi/4}\sec^2(u)\,du$

$u = x^2$

$\frac{du}{dx} = 2x$

$du = 2x\,dx \Rightarrow \frac{1}{2}\,du = x\,dx$

$= \frac{1}{2}\Big[\tan(u)\Big]_0^{\pi/4} = \frac{1}{2}\left(\tan\left(\frac{\pi}{4}\right) - \tan(0)\right)$

$= \frac{1}{2}(1-0) = \boxed{\frac{1}{2}}$

4. $\int_1^3 \frac{3x^2+2x+1}{x^3+x^2+x}\,dx = \int_1^3 \frac{1}{x^3+x^2+x}(3x^2+2x+1)\,dx = \int_{1^3+1^2+1}^{3^3+3^2+3} \frac{1}{u}\,du$

$u = x^3 + x^2 + x$

$\frac{du}{dx} = 3x^2 + 2x + 1$

$du = (3x^2+2x+1)\,dx$

$= \int_3^{39} \frac{1}{u}\,du = \Big[\ln|u|\Big]_3^{39}$

$= \ln|39| - \ln|3| = \ln\left|\frac{39}{3}\right| = \boxed{\ln(13)}$

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1. $\int \sec^2(x^2+x)(2x+1)\,dx = \int \sec^2(u)\,du = \tan(u) + C$

$u = x^2+x$

$\frac{du}{dx} = 2x+1 \Rightarrow du = (2x+1)\,dx$

$= \boxed{\tan(x^2+x) + C}$

2. $\int \frac{\cos(3\ln|x|)}{x}\,dx = \int \cos(3\ln|x|)\frac{1}{x}\,dx = \int \cos(u)\frac{1}{3}\,du$

$u = 3\ln|x|$

$\frac{du}{dx} = 3\,\frac{1}{x}$

$\frac{1}{3}\,du = \frac{1}{x}\,dx$

$= \frac{1}{3}\int \cos(u)\,du$

$= \frac{1}{3}\sin(u) + C = \boxed{\frac{\sin(3\ln|x|)}{3} + C}$

3. $\int_0^3 (x^2-4x+1)^3(2x-4)\,dx = \int_{0^2-4\cdot 0+1}^{3^2-4\cdot 3+1} u^3\,du = \int_1^{-2} u^3\,du$

$u = x^2-4x+1$

$\frac{du}{dx} = 2x-4$

$du = (2x-4)\,dx$

$= \left[\frac{u^4}{4}\right]_1^{-2} = \frac{(-2)^4}{4} - \frac{1^4}{4}$

$= \frac{16}{4} - \frac{1}{4} = \boxed{\frac{15}{4}}$

4. $\int_0^1 \frac{2x+1}{3x^2+3x+1}\,dx = \int_0^1 \frac{1}{3x^2+3x+1}(2x+1)\,dx = \int_{3\cdot 0^2+3\cdot 0+1}^{3\cdot 1^2+3\cdot 1+1} \frac{1}{u}\,\frac{1}{3}\,du$

$u = 3x^2+3x+1$

$\frac{du}{dx} = 6x+3$

$du = (6x+3)\,dx$

$\frac{1}{3}\,du = (2x+1)\,dx$

$= \frac{1}{3}\int_1^7 \frac{1}{u}\,du = \frac{1}{3}\Big[\ln|u|\Big]_1^7$

$= \frac{1}{3}\left(\ln|7| - \ln|1|\right) = \boxed{\frac{1}{3}\ln|7|}$

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1. $\int \sec\left(x^2+x\right)\tan\left(x^2+x\right)(2x+1)\,dx = \int \sec(u)\tan(u)\,du = \sec(u)+C$

$u = x^2 + x$

$\frac{du}{dx} = 2x+1 \implies du = (2x+1)\,dx$

$= \boxed{\sec(x^2+x)+C}$

2. $\int \frac{e^{1/x}}{x^2}\,dx = \int e^{1/x}\frac{1}{x^2}\,dx = \int e^u(-du) = -\int e^u\,du$

$u = \frac{1}{x} = x^{-1}$

$\frac{du}{dx} = -\frac{1}{x^2}$

$-du = \frac{1}{x^2}\,dx$

$= -e^u + C = \boxed{-e^{1/x}+C}$

3. $\int_{-1}^{0} \frac{x}{1+x^2}\,dx = \int_{-1}^{0}\frac{1}{1+x^2}\,x\,dx = \int_{1+(-1)^2}^{1+0^2}\frac{1}{u}\frac{1}{2}\,du$

$u = 1+x^2$

$\frac{du}{dx} = 0+2x$

$du = 2x\,dx \implies \frac{1}{2}du = x\,dx$

$= \frac{1}{2}\int_2^1 \frac{1}{u}\,du = \frac{1}{2}\Big[\ln|u|\Big]_2^1$

$= \frac{1}{2}\left(\ln|1| - \ln|2|\right)$

$= \frac{1}{2}\left(0 - \ln(2)\right) = \boxed{-\frac{\ln(2)}{2}}$

4. $\int_0^{\pi/2} \sin(x)\cos(x)\,dx = \int_0^{\pi/2}(\sin(x))^1\cos(x)\,dx$

$u = \sin(x)$

$\frac{du}{dx} = \cos(x)$

$du = \cos(x)\,dx$

$= \int_{\sin(0)}^{\sin(\pi/2)} u^1\,du = \int_0^1 u^1\,du$

$= \left[\frac{u^2}{2}\right]_0^1 = \frac{1^2}{2} - \frac{0^2}{2} = \boxed{\frac{1}{2}}$