Name: Richard

QUIZ 24 ♡

MATH 200
December 7, 2022

1. $\int_{-1}^{1}(x^3+1)\,dx = \left[\frac{x^4}{4}+x\right]_{-1}^{1} = \left(\frac{1^4}{4}+1\right)-\left(\frac{(-1)^4}{4}+(-1)\right) = \frac{1}{4}+1-\frac{1}{4}+1$

$= \boxed{2}$

2. $\int_0^{\pi}\cos(x)\,dx = \left[\sin(x)\right]_0^{\pi} = \sin(\pi)-\sin(0) = 0-0 = \boxed{0}$

3. Find the area under the graph of $y = e^x$ between $x = 0$ and $x = 1$.

$\int_0^1 e^x = \left[e^x\right]_0^1 = e^1 - e^0 = \boxed{e-1 \text{ square units}}$

4. Find the derivative of the function $F(x) = \int_1^x \frac{\cos(t+2)}{t^3+1}\,dt$.

By FTC 1 $\quad F'(x) = \frac{\cos(x+2)}{x^3+1}$

5. Find the derivative of the function $y = \int_1^{x^2+1} \frac{\cos(t+2)}{t^3+1}\,dt$.

This is $y = F(x^2+1)$ where F is as in (4) above.

By chain rule, $y' = F'(x^2+1)\,2x = \frac{\cos(x^2+1+2)}{(x^2+1)^3+1}\,2x$

$= \boxed{\frac{\cos(x^2+3)}{(x^2+1)^3+1}\,2x}$

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Quiz 24 ♣

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1. $\int_0^2 (x^2+x)\,dx = \left[\frac{x^3}{3}+\frac{x^2}{2}\right] = \left(\frac{2^3}{3}+\frac{2^2}{2}\right)-\left(\frac{0^3}{3}+\frac{0^2}{2}\right)$

$= \frac{8}{3}+\frac{4}{2} = \frac{8}{3}+2 = \frac{8}{3}+\frac{6}{3} = \boxed{\frac{14}{3}}$

2. $\int_0^{\pi/4} \sec^2(x)\,dx = \left[\tan(x)\right]_0^{\pi/4} = \tan(\pi/4) - \tan(0) = 1-0 = \boxed{1}$

3. Find the area under the graph of $y = \frac{1}{x}$ between $x=1$ and $x=e$.

$\int_1^e \frac{1}{x}\,dx = \left[\ln|x|\right]_1^e = \ln|e| - \ln|1| = 1-0$

$= \boxed{1 \text{ square unit}}$

4. Find the derivative of the function $F(x) = \int_1^x \frac{1+e^t}{\sqrt{t+4}}\,dt$.

By FTC 1 $\boxed{F'(x) = \frac{1+e^x}{\sqrt{x+4}}}$

5. Find the derivative of the function $y = \int_1^{x^2+x} \frac{1+e^t}{\sqrt{t+4}}\,dt$.

This is $y = F(x^2+x)$ where F is as in (4) above.

By chain rule, $y' = F'(x^2+x)\,(2x+1)$

$= \boxed{\frac{1+e^{x^2+x}}{\sqrt{x^2+x+4}}(2x+1)}$

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QUIZ 24 ◊

MATH 200
December 7, 2022

1. $\int_{-1}^{1}(x^2+1)\,dx = \left[\frac{x^3}{3}+x\right]_{-1}^{1} = \left(\frac{1^3}{3}+1\right)-\left(\frac{(-1)^3}{3}+(-1)\right) = \frac{1}{3}+1+\frac{1}{3}+1$

$= \frac{2}{3}+2 = \frac{2}{3}+\frac{6}{3} = \boxed{\frac{8}{3}}$

2. $\int_0^1 \sqrt{x}\,dx = \int_0^1 x^{\frac{1}{2}}\,dx = \left[\frac{x^{\frac{1}{2}+1}}{\frac{1}{2}+1}\right]_0^1 = \left[\frac{2}{3}\sqrt{x}^{\,3}\right]_0^1$

$= \frac{2}{3}\sqrt{1}^{\,3} - \frac{2}{3}\sqrt{0}^{\,3} = \frac{2}{3}-0 = \boxed{\frac{2}{3}}$

3. Find the area under the graph of $y=\sin(x)$ between $x=0$ and $x=\pi$.

$\int_0^{\pi}\sin(x)\,dx = \left[-\cos(x)\right]_0^{\pi} = -\cos(\pi)-(-\cos(0)) = -(-1)-(-1)$

$= \boxed{2 \text{ square units}}$

4. Find the derivative of the function $F(x)=\int_1^x \frac{\sqrt{t+4}}{1+\cos(t)}\,dt$.

By FTC 1, $\boxed{F'(x) = \frac{\sqrt{x+4}}{1+\cos(x)}}$

5. Find the derivative of the function $y=\int_1^{\sin(x)} \frac{\sqrt{t+4}}{1+\cos(t)}\,dt$.

This is $y=F(\sin(x))$ where F is as in (4) above.

By chain rule, $D_x\left[F(\sin(x))\right] = F'(\sin(x))\cos(x)$

$= \boxed{\frac{\sqrt{\sin(x)+4}}{1+\cos(\sin(x))}\cos(x)}$

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Quiz 24 ♠

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December 7, 2022

1. $\int_1^2 (x^2+1)\,dx = \left[\frac{x^3}{3} + x\right]_1^2 = \left(\frac{2^3}{3}+2\right) - \left(\frac{1^3}{3}+1\right) = \frac{8}{3} + 2 - \frac{1}{3} - 1$

$= \frac{7}{3} + 1 = \frac{7}{3} + \frac{3}{3} = \boxed{\frac{10}{3}}$

2. $\int_0^\pi \sin(x)\,dx = \left[-\cos(x)\right]_0^\pi = -\cos(\pi) - (-\cos(0)) = -(-1) - (-1) = \boxed{2}$

3. Find the area under the graph of $y = x^2$ between $x = 0$ and $x = 2$.

$\int_0^2 x^2\,dx = \left[\frac{x^3}{3}\right]_0^2 = \frac{2^3}{3} - \frac{0^3}{3} = \boxed{\frac{8}{3} \text{ sq. units.}}$

4. Find the derivative of the function $F(x) = \int_1^x \frac{1+\cos(t)}{\sqrt{t+4}}\,dt$.

By FTC 1, $\boxed{F'(x) = \frac{1+\cos(x)}{\sqrt{x+4}}}$

5. Find the derivative of the function $y = \int_1^{x^2+x} \frac{1+\cos(t)}{\sqrt{t+4}}\,dt$.

This is $y = F(x^2+x)$, where $F(x)$ is as in (4) above.

By chain rule, $D_x\left[F(x^2+x)\right] = F'(x^2+x)(2x+1)$

$= \boxed{\frac{1+\cos(x^2+x)}{\sqrt{x^2+x+4}}(2x+1)}$