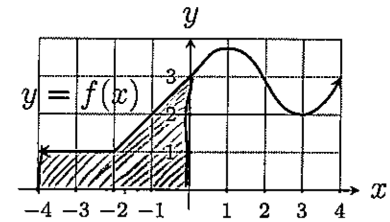
Name: Richard

Quiz 23 ♡

MATH 200
December 5, 2022

1. A function $f(x)$ is graphed below. If $\int_{-4}^{4} f(x)\,dx = 17.8$, what is $\int_{0}^{4} f(x)\,dx$?



$$17.8 = \int_{-4}^{4} f(x)\,dx = \int_{-4}^{0} f(x)\,dx + \int_{0}^{4} f(x)\,dx$$

$$17.8 = (\text{shaded area}) + \int_{0}^{4} f(x)\,dx$$

$$17.8 = 6 + \int_{0}^{4} f(x)\,dx$$

Thus $\int_{0}^{4} f(x)\,dx = 17.8 - 6 = 11.8$

2. Suppose f is a function for which $\int_2^5 f(x)dx = 4$ and $\int_2^8 f(x)dx = 9$. Find $\int_8^5 7f(x)dx$.

Note: $\int_2^8 f(x)\,dx = \int_2^5 f(x)\,dx + \int_5^8 f(x)\,dx \Rightarrow 9 = 4 + \int_5^8 f(x)\,dx$

$$\Rightarrow \boxed{\int_5^8 f(x)\,dx = 5}$$

Then $\int_8^5 7f(x)\,dx = 7\int_8^5 f(x)\,dx = -7\int_5^8 f(x)\,dx = -7 \cdot 5 = \boxed{-35}$

3. Write the limit $\lim\limits_{n\to\infty} \sum_{k=1}^{n} \sin\left(\sqrt{\frac{\pi k}{n}}\right)\frac{\pi}{n}$ as a definite integral.

Let $x_k = \frac{\pi k}{n}$

$x_0 = \frac{\pi \cdot 0}{n} = 0 \leftarrow a$

$x_1 = \frac{\pi \cdot 1}{n}$

$\vdots$

$x_n = \frac{\pi n}{n} = \pi \leftarrow b$

$\Delta x = \frac{b-a}{n} = \frac{\pi - 0}{n} = \frac{\pi}{n}$

$$\lim_{n\to\infty} \sum_{k=1}^{n} \sin\left(\sqrt{\frac{\pi k}{n}}\right)\frac{\pi}{n}$$

$$= \lim_{n\to\infty} \sum_{k=1}^{n} \sin\left(\sqrt{x_k}\right)\Delta x$$

$$= \boxed{\int_0^{\pi} \sin(\sqrt{x})\,dx}$$

4. Write $\int_2^5 \ln(x)\,dx$ as a limit of Riemann sums (such as in problem 3 above).

$$\boxed{\Delta x = \frac{5-2}{n} = \frac{3}{n}}$$

$$x_k = 2 + k\Delta x = 2 + k\frac{3}{n}$$

$$\boxed{x_k = 2 + \frac{3k}{n}}$$

$$\int_2^5 \ln(x)\,dx$$

$$= \boxed{\lim_{n\to\infty} \sum_{k=1}^{n} \ln\left(2 + \frac{3k}{n}\right)\frac{3}{n}}$$

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QUIZ 23 ♣

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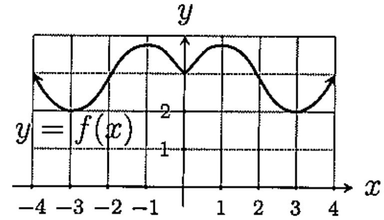
1. A function $f(x)$ is graphed below. If $\int_{-4}^{4} f(x)\,dx = 22.6$, what is $\int_0^4 f(x)\,dx$?

$$22.6 = \int_{-4}^{4} f(x)\,dx = \int_{-4}^{0} f(x)\,dx + \int_0^4 f(x)\,dx$$



But by symmetry, $\int_{-4}^{0} f(x)\,dx = \int_0^4 f(x)\,dx$

and the above becomes $22.6 = 2\int_0^4 f(x)\,dx$

Thus $\int_0^4 f(x)\,dx = \frac{22.6}{2} = \boxed{11.3}$

2. Suppose f and g are functions for which $\int_0^5 f(x)\,dx = 3$, $\int_0^2 3g(x)\,dx = 12$, and $\int_2^5 g(x)\,dx = -1$. Find $\int_0^5 3f(x) - g(x)\,dx$.

Note: $12 = \int_0^2 3\,g(x)\,dx = 3\int_0^2 g(x)\,dx \Longrightarrow \boxed{\int_0^2 g(x)\,dx = \frac{12}{3} = 4}$

Then $\int_0^5 g(x)\,dx = \int_0^2 g(x)\,dx + \int_2^5 g(x)\,dx = 4 - 1 = \boxed{3}$

Now: $\int_0^5 3f(x) - g(x)\,dx = 3\int_0^5 f(x)\,dx - \int_0^5 g(x)\,dx = 3\cdot 3 - 3 = \boxed{6}$

3. $\lim_{n\to\infty} \sum_{k=1}^{n} \frac{1}{1+(2+7k/n)^2}\,\frac{7}{n}$ as a definite integral.

Let $x_k = 2 + \frac{7k}{n}$

$x_0 = 2 + \frac{7\cdot 0}{n} = 2 \longleftarrow a$

$x_1 = 2 + \frac{7\cdot 1}{n}$

$\vdots$

$x_n = 2 + \frac{7\cdot n}{n} = 9 \longleftarrow b$

$\Delta x = \frac{b-a}{n} = \frac{9-2}{n} = \frac{7}{n}$

$$\lim_{n\to\infty} \sum_{k=1}^{n} \frac{1}{1+\left(2+\frac{7k}{n}\right)^2}\,\frac{7}{n}$$

$$= \lim_{n\to\infty} \sum_{k=1}^{n} \frac{1}{1+x_k^2}\,\Delta x$$

$$= \boxed{\int_2^9 \frac{1}{1+x^2}\,dx}$$

Write $\int_3^4 \sin(x)\,dx$ as a limit of Riemann sums (such as in problem 3 above).

$\Delta x = \frac{4-3}{n} = \frac{1}{n}$

$x_k = 3 + k\cdot\Delta x = 3 + \frac{k}{n}$

$$\int_3^4 \sin(x)\,dx = \lim_{n\to\infty} \sum_{k=1}^{n} \sin(x_k)\,\Delta x = \boxed{\lim_{n\to\infty} \sum_{k=1}^{n} \sin\left(3+\frac{k}{n}\right)\frac{1}{n}}$$

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QUIZ 23 ◊

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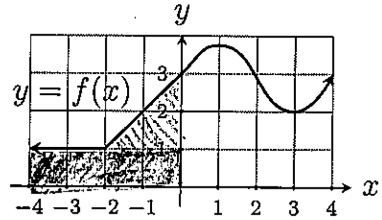

$$17.8 = \int_{-4}^{4} f(x)\,dx = \int_{-4}^{0} f(x)\,dx + \int_0^4 f(x)\,dx$$

$$17.8 = [\text{shaded area}] + \int_0^4 f(x)\,dx$$

$$17.8 = 6 + \int_0^4 f(x)\,dx \implies \int_0^4 f(x)\,dx = \boxed{11.8}$$

2. Suppose f is a function for which $\int_2^5 f(x)dx = 4$ and $\int_2^8 f(x)dx = 9$. Find $\int_8^5 7f(x)dx$.

$$\int_2^8 f(x)\,dx = \int_2^5 f(x)\,dx + \int_5^8 f(x)\,dx$$

$$9 = 4 + \int_5^8 f(x)\,dx \implies \boxed{\int_5^8 f(x)\,dx = 5}$$

Now $\int_8^5 7f(x)\,dx = 7\int_8^5 f(x)\,dx = -7\int_5^8 f(x)\,dx = -7.5 = \boxed{-35}$

3. Write the limit $\lim\limits_{n\to\infty} \sum\limits_{k=1}^{n} \sin\left(\sqrt{\frac{\pi k}{n}}\right)\frac{\pi}{n}$ as a definite integral.

$x_k = \frac{\pi k}{n}$

k	$x_k = \frac{\pi k}{n}$
0	$x_0 = \frac{\pi \cdot 0}{n} = 0 \leftarrow a$
1	$x_1 = \frac{\pi \cdot 1}{n}$
⋮	
n	$x_n = \frac{\pi \cdot n}{n} = \pi \leftarrow b$

$$\Delta x = \frac{b-a}{n} = \frac{\pi - 0}{n} = \frac{\pi}{n}$$

$$\lim_{n\to\infty} \sum_{k=1}^{n} \sin\left(\sqrt{\frac{\pi k}{n}}\right)\frac{\pi}{n}$$

$$\lim_{n\to\infty} \sum_{k=1}^{n} \sin(\sqrt{x_k})\Delta x$$

$$= \boxed{\int_0^{\pi} \sin(\sqrt{x})\,dx}$$

4. Write $\int_0^5 e^x\,dx$ as a limit of Riemann sums (such as in problem 3 above).

$$\Delta x = \frac{5-0}{n} = \frac{5}{n}$$

$$x_k = 0 + k\Delta x = \frac{5k}{n}$$

$$\int_0^5 e^x\,dx = \lim_{n\to\infty} \sum_{k=1}^{n} e^{5k/n} \cdot \frac{5}{n}$$

$$= \lim_{n\to\infty} \sum_{k=1}^{n} \frac{5e^{5k/n}}{n}$$

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QUIZ 23 ♠

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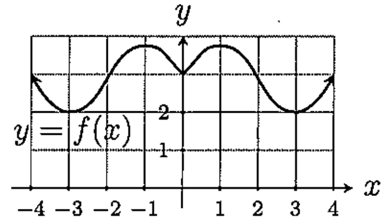
1. A function $f(x)$ is graphed below. If $\int_{-4}^{4} f(x)\,dx = 22.6$, what is $\int_0^4 f(x)\,dx$?



By symmetry, $\int_{-4}^{0} f(x)\,dx = \int_0^4 f(x)\,dx$, so

$$\underline{22.6} = \int_{-4}^{4} f(x)\,dx = \int_{-4}^{0} f(x)\,dx + \int_0^4 f(x)\,dx = \underline{2\int_0^4 f(x)\,dx}$$

Thus $\int_0^4 f(x)\,dx = \dfrac{22.6}{2} = \boxed{11.3}$

2. Suppose f and g are functions for which $\int_0^5 f(x)\,dx = 3$, $\int_0^2 3g(x)\,dx = 12$, and $\int_2^5 g(x)\,dx = -1$. Find $\int_0^5 3f(x) - g(x)\,dx$.

$$12 = \int_0^2 3g(x)\,dx \Longrightarrow 12 = 3\int_0^2 g(x)\,dx \Rightarrow \int_0^2 g(x)\,dx = \frac{12}{3} = \boxed{4}$$

Now, $\int_0^5 3f(x) - g(x)\,dx = \int_0^5 3f(x)\,dx - \int_0^5 g(x)\,dx = 3\int_0^5 f(x)\,dx - \int_0^5 g(x)\,dx$

$$= 3\cdot 3 - \left(\int_0^2 g(x)\,dx + \int_2^5 g(x)\,dx\right) = 9 - (4-1) = \boxed{6}$$

3. $\lim\limits_{n\to\infty} \sum\limits_{k=1}^{n} \dfrac{1}{1+(7k/n)^2}\dfrac{7}{n}$ as a definite integral. $= \lim\limits_{n\to\infty} \sum\limits_{k=1}^{n} \dfrac{1}{1+x_k^2}\Delta x = \boxed{\int_0^7 \frac{1}{1+x^2}\,dx}$

K	7K/n
0	0 ← a
1	7·1/n
2	7·2/n
⋮	⋮
n	7·n/n = 7 ← b

$$\Delta x = \frac{b-a}{n} = \frac{7-0}{n} = \frac{7}{n}$$

Write $\int_3^4 \sqrt{x}\,dx$ as a limit of Riemann sums (such as in problem 3 above).

$$\Delta x = \frac{4-3}{n} = \frac{1}{n}$$

$$x_k = 3 + k\Delta x = 3 + \frac{k}{n}$$

$$\int_3^4 \sqrt{x}\,dx = \boxed{\lim_{n\to\infty} \sum_{k=1}^{n} \sqrt{3+\frac{k}{n}}\cdot\frac{1}{n}}$$