Name: Richard

QUIZ 22 ♡

MATH 200
November 30, 2022

1. (6 points) $\int (x + \sin^2(x) + \cos^2(x))\, dx = \int (x+1)\,dx = \boxed{\frac{x^2}{2} + x + C}$

(Because $\sin^2(x) + \cos^2(x) = 1$.)

2. (7 points) Suppose $f(x)$ is a function for which $f'(x) = 3x^2 + 1$. The graph of f passes through the point $(1, 3)$. Find $f(x)$.

$f(x) = \int (3x^2+1)\,dx = 3\frac{x^3}{3} + x + C$

$f(x) = x^3 + x + C$

Know $3 = f(1) = 1^3 + 1 + C$

$3 = 2 + C$

$C = 1$

$f(x) = x^3 + x + 1$

3. (7 points) What constant acceleration will cause a car to increase its velocity from 20 feet per second to 25 feet per second in 10 seconds?

Let the constant (presently unknown) acceleration be a f/s^2

Then $V(t) = \int a\,dt = at + c$.

It is given that $V(0) = 20$ f/s and $V(10) = 25$ f/s

In particular $20 = V(0) = a \cdot 0 + C$ so $\boxed{C = 20}$

and $\boxed{V(t) = at + 20}$

But also $25 = V(10) = a \cdot 10 + 20$, i.e. $25 = 10a + 20$

so $10a = 5$, so $a = 5/10 = 1/2$

Answer: Acceleration is $\boxed{1/2 \text{ ft/sec/sec}}$

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MATH 200

November 30, 2022

1. (6 points) $\int \sqrt{x}\,(1+x^2)\,dx = \int x^{\frac{1}{2}}(1+x^2)\,dx = \int \left(x^{\frac{1}{2}} + x^{\frac{1}{2}}x^2\right)dx$

$= \int \left(x^{\frac{1}{2}} + x^{5/2}\right)dx = \dfrac{x^{\frac{1}{2}+1}}{\frac{1}{2}+1} + \dfrac{x^{5/2+1}}{5/2+1} + C$

$= \dfrac{x^{3/2}}{3/2} + \dfrac{x^{7/2}}{7/2} + C = \boxed{\frac{2}{3}\sqrt{x}^{\,3} + \frac{2}{7}\sqrt{x}^{\,7} + C}$

2. (7 points) Suppose $f(x)$ is a function for which $f'(x) = \dfrac{8}{x^3} + x$. The graph of f passes through the point $(2, 10)$. Find $f(x)$.

$f(x) = \int f'(x)\,dx = \int \left(\frac{8}{x^3} + x\right)dx = \int \left(8x^{-3} + x\right)dx$

$= 8\dfrac{x^{-3+1}}{-3+1} + \dfrac{x^2}{2} + C = -4x^{-2} + \dfrac{x^2}{2} + C$

$\boxed{f(x) = \frac{x^2}{2} - \frac{4}{x^2} + C}$

Know $10 = f(2) = \dfrac{2^2}{2} - \dfrac{4}{2^2} + C = 2 - 1 + C = 1 + C$

Thus $10 = 1 + C$, so $\boxed{C = 9}$ Answer: $\boxed{f(x) = \frac{x^2}{2} - \frac{4}{x^2} + 9}$

3. (7 points) A rock is dropped from a 1600 foot tall building, with an initial velocity of 0 feet per second. The acceleration due to gravity is -32 feet per second per second. How long does it take the for the rock to strike the ground?

$v(t) = \int a(t)\,dt = \int -32\,dt = -32t + C$

But $0 = v(0) = -32 \cdot 0 + C$, so $C = 0$ and $\boxed{v(t) = -32t}$

Then $s(t) = \int v(t)\,dt = \int -32t\,dt = -16t^2 + C$

Know $1600 = s(0) = -16 \cdot 0^2 + C$, so $C = 1600$

Thus $\boxed{s(t) = -16t^2 + 1600.}$ Rock hits ground when $s(t) = 0 \Rightarrow -16t^2 + 1600 = 0 \Rightarrow 16t^2 = 1600$

$\Rightarrow t^2 = 100 \Rightarrow t = \sqrt{100} = 10$ $\boxed{\text{Answer } 10 \text{ seconds}}$

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1. (6 points) $\int \frac{\sqrt{x}+1}{\sqrt{x}}\,dx = \int\left(\frac{\sqrt{x}}{\sqrt{x}} + \frac{1}{\sqrt{x}}\right)dx = \int\left(1+x^{-1/2}\right)dx$

$= x + \frac{x^{-\frac{1}{2}+1}}{-\frac{1}{2}+1} + C = x + \frac{x^{1/2}}{1/2} + C = \boxed{x + 2\sqrt{x} + C}$

2. (7 points) Suppose $f(x)$ is a function for which $f'(x) = 2x+\cos(x)$ and its graph passes through the point $(\pi, 2)$. Find $f(x)$.

$f(x) = \int f'(x)\,dx = \int(2x+\cos(x))\,dx = \boxed{x^2+\sin(x)+C}$

But also $2 = f(\pi) = \pi^2 + \sin(\pi) + C = \pi^2 + 0 + C$

and hence $\boxed{C = 2-\pi^2}$

Answer $\boxed{f(x) = x^2 + \sin(x) + 2 - \pi^2}$

3. (7 points) A stone is thrown vertically upward with an initial velocity of 8 feet per second. Assuming the acceleration due to gravity is -32 feet per second per second, how long does it take the stone to stop rising?

$v(t) = \int a(t)\,dt = \int -32\,dt = \boxed{-32t + C}$

But also $8 = v(0) = -32\cdot 0 + C$, so $\boxed{C = 8}$

and this $\boxed{v(t) = -32t + 8}$

Stone stops rising when $v(t) = 0$

$-32t + 8 = 0$

$32t = 8$

$t = \frac{8}{32} = \frac{1}{4}$

Answer Stops rising at $t = 1/4$ second

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QUIZ 22 ♠

MATH 200
November 30, 2022

1. (6 points) $\int (3-x)^2\,dx = \int (3-x)(3-x)\,dx = \int (9-6x+x^2)\,dx$

$$= 9x - 6\frac{x^2}{2} + \frac{x^3}{3} + C = \boxed{9x - 3x^2 + \frac{x^3}{3} + C}$$

2. (7 points) Suppose $f(x)$ is a function for which $f'(x) = \sqrt{x} + 2$ and $f(4) = 7$. Find $f(x)$.

$$f(x) = \int f'(x)\,dx = \int (x^{\frac{1}{2}} + 2)\,dx = \frac{x^{\frac{1}{2}+1}}{\frac{1}{2}+1} + 2x + C$$

$$= \frac{x^{3/2}}{3/2} + 2x + C = \boxed{\frac{2\sqrt{x}^3}{3} + 2x + C}$$

Know $7 = f(4) = \frac{2\sqrt{4}^3}{3} + 2 \cdot 4 + C = \frac{16}{3} + 8 + C$

So $C = 7 - \frac{16}{3} - 8 = -1 - \frac{16}{3} = -\frac{3}{3} - \frac{16}{3} = \boxed{\frac{-19}{3}}$

$\boxed{f(x) = \frac{2\sqrt{x}^3}{3} + 2x - \frac{19}{3}}$ ← Answer

3. (7 points) A freight train travels on a straight track with a constant acceleration. At time $t = 0$ its velocity is 10 miles per hour. Half an hour later (at $t = 0.5$ hours) it is traveling at 70 mph. How far did it travel in the half hour period?

Let the constant (unknown) acceleration be $a\ m/h^2$

Then velocity $= \int a\,dt = at + C$

Thus $v(t) = at + c$, but we know $10 = v(0) = a \cdot 0 + c$, hence $c = 10$, and $v(t) = at + 10$. Also we know $70 = v(1/2) = a \cdot \frac{1}{2} + 10$, so $60 = \frac{a}{2}$ making $a = 120$. Hence $\boxed{v(t) = 120t + 10}$

Now, $s(t) = \int v(t)\,dt = \int (120t + 10)\,dt = 60t^2 + 10t + C$

But $0 = s(0) = 60 \cdot 0^2 + 10 \cdot 0 + C$, so $C = 0$ and $\boxed{s(t) = 60t^2 + 10t}$

Answer: at time $t = 1/2$ $s(1/2) = 60(\frac{1}{2})^2 + 10 \cdot \frac{1}{2} = \boxed{20 \text{ miles}}$