Name: Richard

Quiz 21 ♡

MATH 200
November 29, 2021

1. $\lim_{x \to 0^+} (e^x - 1)^x = \lim_{x \to 0^+} e^{\ln((e^x-1)^x)} = \lim_{x \to 0^+} e^{x \ln(e^x - 1)}$

form 0^0

$= e^{\lim_{x \to 0^+} x \ln(e^x - 1)} = e^{\lim_{x \to 0^+} \frac{\ln(e^x-1)}{1/x}}$

form $0 \cdot \infty$

form $\frac{\infty}{\infty}$

$= e^{\lim_{x \to 0^+} \frac{\frac{e^x}{e^x - 1}}{-1/x^2}} = e^{\lim_{x \to 0^+} \frac{-x^2 e^x}{e^x - 1}}$

Now form $\frac{0}{0}$!

$= e^{\lim_{x \to 0^+} \frac{-2x e^x - x^2 e^x}{e^x - 0}} = e^{\lim_{x \to 0^+} \frac{-e^x(2x + x^2)}{e^x}}$

$= e^{\lim_{x \to 0^+} -(2x + x^2)} = e^{-(2 \cdot 0 + 0^2)} = e^0 = \boxed{1}$

2. $\int \left(x^3 + \frac{1}{x} + \sin(x)\right) dx = \frac{1}{3+1} x^{3+1} + \ln|x| - \cos(x) + C$

$= \boxed{\frac{x^4}{4} + \ln|x| - \cos(x) + C}$

3. $\int \sqrt[3]{x}^4 \, dx = \int x^{\frac{4}{3}} dx = \frac{1}{\frac{4}{3}+1} x^{\frac{4}{3}+1} + C = \frac{1}{7/3} x^{7/3} + C$

$= \frac{3}{7} x^{7/3} + C = \boxed{\frac{3}{7} \sqrt[3]{x}^7 + C}$

4. $\int \sec(x) \tan(x) \, dx = \boxed{\sec(x) + C}$

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1. $\lim_{x \to 0^+} x^x = \lim_{x\to 0^+} e^{\ln(x^x)} = \lim_{x \to 0^+} e^{x \ln(x)}$

form 0^0

$= e^{\lim_{x\to 0^+} x \ln(x)}$

form $0 \cdot \infty$

$= e^{\lim_{x\to 0^+} \frac{\ln(x)}{1/x}}$

form $\frac{\infty}{\infty}$

$= e^{\lim_{x\to 0^+} \frac{1/x}{-1/x^2}}$

$= e^{\lim_{x\to 0^+} -x} = e^0 = \boxed{1}$

2. $\int \left(x^5 - \frac{1}{x^2} + \cos(x)\right) dx = \int (x^5 - x^{-2} + \cos(x))\, dx$

$= \frac{1}{5+1} x^{5+1} - \frac{1}{-2+1} x^{-2+1} + \sin(x) + C$

$= \boxed{\frac{x^6}{6} + \frac{1}{x} + \sin(x) + C}$

3. $\int \sqrt[3]{x}\, dx = \int x^{\frac{1}{3}} dx = \frac{1}{\frac{1}{3}+1} x^{\frac{1}{3}+1} + C = \frac{1}{4/3} x^{4/3} + C$

$= \boxed{\frac{3}{4} \sqrt[3]{x}^4 + C}$

4. $\int \frac{1}{1+x^2}\, dx = \boxed{\tan^{-1}(x) + C}$

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1. $\lim_{x\to\infty} (\ln x)^{1/x} = \lim_{x\to\infty} e^{\ln((\ln(x))^{1/x})}$

form ∞^0

$$= \lim_{x\to\infty} e^{\frac{1}{x}\ln(\ln(x))} = \lim_{x\to\infty} e^{\frac{\ln(\ln(x))}{x}}$$

$$= e^{\lim_{x\to\infty} \frac{\ln(\ln(x))}{x}} = e^{\lim_{x\to\infty} \frac{\frac{1}{\ln(x)}\frac{1}{x}}{1}}$$

form $\frac{\infty}{\infty}$

$$= e^{\lim_{x\to\infty} \frac{1}{x\ln(x)}} = e^0 = \boxed{1}$$

2. $\int \left(5x - \frac{1}{x} + \sec^2(x)\right) dx = \int \left(5x^1 - \frac{1}{x} + \sec^2(x)\right) dx$

$$= 5\frac{1}{1+1}x^{1+1} - \ln|x| + \tan(x) + C$$

$$= \boxed{\frac{5}{2}x^2 - \ln|x| + \tan(x) + C}$$

3. $\int \frac{1}{\sqrt{x}}\, dx = \int x^{-1/2}\, dx = \frac{1}{-\frac{1}{2}+1}x^{-\frac{1}{2}+1} + C = \frac{1}{\frac{1}{2}}x^{1/2} + C$

$$= \boxed{2\sqrt{x} + C}$$

4. $\int \frac{1}{\sqrt{1-x^2}}\, dx = \boxed{\sin^{-1}(x) + C}$

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1. $$\lim_{x\to\infty}\left(1+\frac{1}{x}\right)^x = \lim_{x\to\infty} e^{\ln\left(\left(1+\frac{1}{x}\right)^x\right)} = \lim_{x\to\infty} e^{x\ln\left(1+\frac{1}{x}\right)}$$

form 1^∞

$$= e^{\lim\limits_{x\to\infty} x\ln\left(1+\frac{1}{x}\right)} = e^{\lim\limits_{x\to\infty} \frac{\ln\left(1+\frac{1}{x}\right)}{\frac{1}{x}}}$$

form $\infty\cdot 0$

form $\frac{0}{0}$

$$= e^{\lim\limits_{x\to\infty} \frac{\frac{1}{1+\frac{1}{x}}\left(0-\frac{1}{x^2}\right)}{-\frac{1}{x^2}}} = e^{\lim\limits_{x\to\infty} \frac{1}{1+\frac{1}{x}}} = e^{\frac{1}{1+0}} = e^1 = \boxed{e}$$

2. $$\int \left(2+x-x^2+\csc^2(x)\right)dx = \int\left(2+x^1-x^2+\csc^2(x)\right)dx$$
$$= 2x+\frac{1}{1+1}x^{1+1}-\frac{1}{2+1}x^{2+1}-\cot(x)+C$$
$$= \boxed{2x+\frac{1}{2}x^2-\frac{1}{3}x^3-\cot(x)+C}$$

3. $$\int \sqrt{x}\,dx = \int x^{\frac{1}{2}}\,dx$$
$$= \frac{1}{\frac{1}{2}+1}x^{\frac{1}{2}+1}+C = \frac{1}{\frac{3}{2}}x^{\frac{3}{2}}+C = \boxed{\frac{2}{3}\sqrt{x}^{\,3}+C}$$

4. $$\int \csc(x)\cot(x)\,dx = \boxed{-\csc(x)+C}$$