

1. $\lim_{x \rightarrow 1} \frac{\ln|x|}{x-1} = \lim_{x \rightarrow 1} \frac{\frac{1}{x}}{1-0} = \frac{\frac{1}{1}}{1} = \boxed{1}$

↑
form $\frac{0}{0}$

2. $\lim_{x \rightarrow \infty} \frac{1+e^x}{e^x-1} = \lim_{x \rightarrow \infty} \frac{0+e^x}{e^x-0} = \lim_{x \rightarrow \infty} \frac{e^x}{e^x} = \lim_{x \rightarrow \infty} 1 = \boxed{1}$

↑
form $\frac{\infty}{\infty}$

3. $\lim_{x \rightarrow \pi/2} \left(\frac{\pi}{2} - x\right) \sec(x) = \lim_{x \rightarrow \pi/2} \frac{\left(\frac{\pi}{2} - x\right)}{\frac{1}{\sec(x)}} = \lim_{x \rightarrow \pi/2} \frac{\left(\frac{\pi}{2} - x\right)}{\cos(x)}$

↑
form $0 \cdot \infty$

↑
form $\frac{0}{0}$

$= \lim_{x \rightarrow \pi/2} \frac{0-1}{-\sin(x)} = \frac{-1}{-\sin(\pi/2)} = \frac{-1}{-1} = \boxed{1}$

4. $\lim_{x \rightarrow 0} \left(\frac{1}{x} - \frac{1}{e^x-1}\right) = \lim_{x \rightarrow 0} \left(\frac{1}{x} \frac{e^x-1}{e^x-1} - \frac{x}{x} \frac{1}{e^x-1}\right)$

$= \lim_{x \rightarrow 0} \frac{e^x-1-x}{x(e^x-1)} = \lim_{x \rightarrow 0} \frac{e^x-0-1}{(e^x-1)+xe^x}$

↑
form $\frac{0}{0}$

↑
still form $\frac{0}{0}$

$= \lim_{x \rightarrow 0} \frac{e^x}{e^x+e^x+xe^x} = \frac{e^0}{e^0+e^0+0e^0} = \frac{1}{1+1+0} = \boxed{\frac{1}{2}}$

$$1. \lim_{x \rightarrow \pi} \frac{\cos(x) + 1}{(x - \pi)^2} = \lim_{x \rightarrow \pi} \frac{-\sin(x) + 0}{2(x - \pi)} = \lim_{x \rightarrow \pi} \frac{-\cos(x)}{2}$$

form $\frac{0}{0}$ still form $\frac{0}{0}$ $= \frac{-\cos(\pi)}{2} = \frac{-(-1)}{2} = \boxed{\frac{1}{2}}$

$$2. \lim_{x \rightarrow 0} \frac{4 + 2 \ln|x|}{5 - 3 \ln|x|} = \lim_{x \rightarrow 0} \frac{0 + 2 \cdot \frac{1}{x}}{0 - 3 \cdot \frac{1}{x}} = \lim_{x \rightarrow 0} \frac{2 \cdot \frac{1}{x}}{-3 \cdot \frac{1}{x}} = \lim_{x \rightarrow 0} \frac{-2}{3} = \boxed{\frac{-2}{3}}$$

form $\frac{\infty}{\infty}$

$$3. \lim_{x \rightarrow \infty} x e^{-x} = \lim_{x \rightarrow \infty} \frac{x}{\frac{1}{e^{-x}}} = \lim_{x \rightarrow \infty} \frac{x}{e^x} = \lim_{x \rightarrow \infty} \frac{1}{e^x} = \boxed{0}$$

form $\infty \cdot 0$ form $\frac{\infty}{\infty}$

$$4. \lim_{x \rightarrow \infty} (\ln(x) - \ln(x+1)) = \lim_{x \rightarrow \infty} \ln\left(\frac{x}{x+1}\right) = \ln\left(\lim_{x \rightarrow \infty} \frac{x}{x+1}\right)$$

$$= \ln\left(\lim_{x \rightarrow \infty} \frac{1}{1 + \frac{1}{x}}\right) = \ln(1) = \boxed{0}$$

form $\frac{\infty}{\infty}$

$$1. \lim_{x \rightarrow 1} \frac{\ln|x|}{4x - x^2 - 3} = \lim_{x \rightarrow 1} \frac{\frac{1}{x}}{4 - 2x} = \frac{\frac{1}{1}}{4 - 2 \cdot 1} = \boxed{\frac{1}{2}}$$

form $\frac{0}{0}$

$$2. \lim_{x \rightarrow \pi/2} \frac{1 + \tan(x)}{1 - 3 \tan(x)} = \lim_{x \rightarrow \pi/2} \frac{0 + \sec^2(x)}{0 - 3 \sec^2(x)} = \lim_{x \rightarrow \pi/2} -\frac{1}{3} \frac{\sec^2(x)}{\sec^2(x)}$$

$$= \lim_{x \rightarrow \pi/2} -\frac{1}{3} = \boxed{-\frac{1}{3}}$$

form $\frac{\infty}{\infty}$

$$3. \lim_{x \rightarrow \infty} x \sin\left(\frac{1}{4x}\right) = \lim_{x \rightarrow \infty} \frac{\sin\left(\frac{1}{4x}\right)}{\frac{1}{x}} = \lim_{x \rightarrow \infty} \frac{\cos\left(\frac{1}{4x}\right) \left(\frac{-1}{4x^2}\right)}{-\frac{1}{x^2}}$$

$$= \lim_{x \rightarrow \infty} \frac{1}{4} \cos\left(\frac{1}{4x}\right)$$

$$= \frac{1}{4} \cos(0) = \boxed{\frac{1}{4}}$$

form $\infty \cdot 0$

form $\frac{0}{0}$

$$4. \lim_{x \rightarrow \infty} (\ln(x) - \ln(x+1)) =$$

$$= \lim_{x \rightarrow \infty} \ln\left(\frac{x}{x+1}\right) = \ln\left(\lim_{x \rightarrow \infty} \frac{x}{x+1}\right)$$

$$= \ln\left(\lim_{x \rightarrow \infty} \frac{1}{1+\frac{1}{x}}\right) = \ln(1) = \boxed{0}$$

form $\frac{\infty}{\infty}$

$$1. \lim_{x \rightarrow 1} \frac{e^x - e}{x^2 - 1} = \lim_{x \rightarrow 1} \frac{e^x - 0}{2x - 0} = \frac{e^1}{2 \cdot 1} = \boxed{\frac{e}{2}}$$

↑
form $\frac{0}{0}$

$$2. \lim_{x \rightarrow \infty} \frac{\ln|x|}{\sqrt{x}} = \lim_{x \rightarrow \infty} \frac{\frac{1}{x}}{\frac{1}{2\sqrt{x}}} = \lim_{x \rightarrow \infty} \frac{2\sqrt{x}}{x} = \lim_{x \rightarrow \infty} \frac{2\sqrt{x}}{\sqrt{x}\sqrt{x}}$$

↑
form $\frac{\infty}{\infty}$

$$= \lim_{x \rightarrow \infty} \frac{2}{\sqrt{x}} = \boxed{0}$$

$$3. \lim_{x \rightarrow 0} x \csc(x) = \lim_{x \rightarrow 0} \frac{x}{\frac{1}{\csc(x)}} = \lim_{x \rightarrow 0} \frac{x}{\sin(x)} = \lim_{x \rightarrow 0} \frac{1}{\cos(x)}$$

↑
form $0 \cdot \infty$

↑
form $\frac{0}{0}$

$$= \frac{1}{\cos(0)} = \boxed{1}$$

$$4. \lim_{x \rightarrow \infty} (2 \ln(x) - \ln(x^2 + 1)) = \lim_{x \rightarrow \infty} (\ln(x^2) - \ln(x^2 + 1))$$

↑
form $\infty - \infty$

$$= \lim_{x \rightarrow \infty} \ln\left(\frac{x^2}{x^2 + 1}\right) = \ln\left(\lim_{x \rightarrow \infty} \frac{x^2}{x^2 + 1}\right)$$

$$= \ln\left(\lim_{x \rightarrow \infty} \frac{2x}{2x + 0}\right) = \ln\left(\lim_{x \rightarrow \infty} \frac{2}{2}\right) = \ln(1) = \boxed{0}$$

↑
form $\frac{\infty}{\infty}$

↑
form $\frac{\infty}{\infty}$