Name: ______________________________

Quiz 19

MATH 200
April 7, 2022

1. Consider $f(x) = x + \sin(x)$ on $[0, 2\pi]$. Find *all* numbers c in $(0, 2\pi)$ guaranteed by the mean value theorem.

We seek all values x for which

$$\begin{aligned}
f'(x) &= \frac{f(2\pi) - f(0)}{2\pi - 0} \\
1 + \cos(x) &= \frac{\big(2\pi + \sin(2\pi)\big) - \big(0 + \sin(0)\big)}{2\pi} \\
1 + \cos(x) &= \frac{\big(2\pi + 0\big) - \big(0 + 0\big)}{2\pi} \\
1 + \cos(x) &= \frac{2\pi}{2\pi} \\
\cos(x) &= 0
\end{aligned}$$

The values of x in $(0, 2\pi)$ for which $\cos(x) = 0$ are $x = \frac{\pi}{2}$ and $x = \frac{3\pi}{2}$.

Answer: $\boxed{c = \frac{\pi}{2} \text{ and } c = \frac{3\pi}{2}}$

2. Find the linear approximation $L(x)$ for the function $f(x) = x + \frac{3}{x}$ at $x = 3$.

The derivative is $f'(x) = 1 - \frac{3}{x^2}$.

$$\begin{aligned}
L(x) &= f(3) + f'(3)(x - 3) \\
&= 3 + \frac{3}{3} + \left(1 - \frac{3}{3^2}\right)(x - 3) \\
&= 4 + \left(1 - \frac{1}{3}\right)(x - 3) \\
&= 4 + \frac{2}{3}(x - 3) \\
&= 4 + \frac{2}{3}x - 2 \\
&= \frac{2}{3}x + 2
\end{aligned}$$

Answer: $\boxed{L(x) = \frac{2}{3}x + 2}$

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Quiz 19

MATH 200
April 7, 2022

1. Consider $f(x) = 2 + \cos(x)$ on $[0, 2\pi]$. Find *all* numbers c in $(0, 2\pi)$ guaranteed by the mean value theorem.

We seek all values x for which

$$\begin{aligned} f'(x) &= \frac{f(2\pi) - f(0)}{2\pi - 0} \\ 0 - \sin(x) &= \frac{\big(2 + \cos(2\pi)\big) - \big(2 + \cos(0)\big)}{2\pi} \\ -\sin(x) &= \frac{0}{2\pi} \\ -\sin(x) &= 0 \end{aligned}$$

The only value of x in $(0, 2\pi)$ for which $-\sin(x) = 0$ is $x = \pi$.

Answer: $\boxed{c = \pi}$

2. Find the linear approximation $L(x)$ for the function $f(x) = 2e^x - x$ at $x = \ln(3)$.

The derivative is $f'(x) = 2e^x - 1$.

$$\begin{aligned} L(x) &= f(\ln(3)) + f'(\ln(3))(x - 0) \\ &= 2e^{\ln(3)} - \ln(3) + (2e^{\ln(3)} - 1)(x - \ln(3)) \\ &= 2 \cdot 3 - \ln(3) + (2 \cdot 3 - 1)(x - \ln(3)) \\ &= 6 - \ln(3) + 5(x - \ln(3)) \\ &= 6 - \ln(3) + 5x - 5\ln(3) \\ &= 5x + 6 - 6\ln(3) \end{aligned}$$

Answer: $\boxed{L(x) = 5x + 6 - 6\ln(3)}$