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QUIZ 19

MATH 200
November 2, 2023

1. (10 pts.) Is it possible to have a continuous and differentiable function f that satisfies the following:

- $f(0) = -1$
- $f(2) = 4$
- $f'(x) \leq 2$ for all values of x.

If so, sketch a graph of such a function f. If not, justify why not.

Such a function would be continuous on $[0,2]$ and differentiable on $(0,2)$, so by the Mean Value Theorem There would be a number c in $(0,2)$ for which

$$f'(c) = \frac{f(2)-f(0)}{2-0} = \frac{4-(-1)}{2} = \frac{5}{2} = 2.5$$

However, this is impossible, because, as stated, $f'(x) \leq 2$ for all x, but for $x=c$ $f'(x) > 2$.

Answer $\boxed{\text{It is not possible to have such a function}}$

2. (10 pts.) Information about a function f and its derivative is given in the table below.

(a) Find the linear approximation $L(x)$ of $f(x)$ at $x = 4$.
Write your answer in the form $L(x) = mx + b$.

x	0	2	4	6	8	10
$f(x)$	-3	-2	1	6	6	3
$f'(x)$	5	3	2	1	0	-2

$$L(x) = f(a) + f'(a)(x-a)$$
$$L(x) = f(4) + f'(4)(x-4)$$
$$= 1 + 2(x-4)$$
$$= 1 + 2x - 8$$
$$= 2x - 7$$

$$\boxed{L(x) = 2x - 7}$$

(b) Use this to find an approximate value of $f(4.5)$.

$$f(4.5) \approx L(4.5) = 2 \cdot 4.5 - 7 = 9 - 7 = 2$$

$$\therefore \boxed{f(4.5) \approx 2}$$