Name: Richard

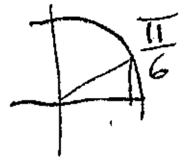
QUIZ 17

MATH 200
October 26, 2023

1. Find the global extrema of the function $f(x) = \dfrac{x}{2} + \cos(x)$ on the closed interval $\left[0, \dfrac{\pi}{2}\right]$.

$$f'(x) = \frac{1}{2} - \sin(x) = 0$$

$$\sin(x) = \frac{1}{2}$$



$$x = \frac{\pi}{6} \longleftarrow \text{critical point}$$

$$f(0) = \frac{0}{2} + \cos(0) = 1$$

$$f\left(\frac{\pi}{6}\right) = \frac{\pi/6}{2} + \cos\left(\frac{\pi}{6}\right) = \frac{\pi}{12} + \frac{\sqrt{3}}{2} = \frac{\pi + 6\sqrt{3}}{12} \approx \frac{3.1 + 6 \cdot 1.7}{12} = \frac{13.3}{2} > 1$$

$$f\left(\frac{\pi}{2}\right) = \frac{\pi/2}{2} + \cos\left(\frac{\pi}{2}\right) = \frac{\pi}{4} \approx \frac{3.14}{4} \approx 0.78$$

f has a global max of $f\left(\frac{\pi}{6}\right) = \frac{\pi + 6\sqrt{3}}{12}$ at $x = \frac{\pi}{6}$

f has a global min of $f\left(\frac{\pi}{2}\right) = \frac{\pi}{4}$ at $x = \frac{\pi}{2}$

2. Find the global extrema of the function $f(x) = 8\sqrt{x} - x$ on the open interval $(0, 25)$.

$$f(x) = 8x^{\frac{1}{2}} - x$$

$$f'(x) = 8 \cdot \frac{1}{2} x^{-\frac{1}{2}} - 1 = \frac{4}{\sqrt{x}} - 1 = 0$$

Only one critical point on the open interval!

$$\frac{4}{\sqrt{x}} = 1$$

$$4 = \sqrt{x}$$

$$\boxed{x = 16} \longleftarrow \text{critical point}$$

$$f''(x) = -2x^{-\frac{3}{2}} = \frac{-2}{\sqrt{x}^{\,3}}$$

$$f''(16) = -\frac{2}{\sqrt{16}^{\,3}} = -\frac{2}{4^3} < 0 \longleftarrow \text{so local max at } x = 16$$

f has a global max of $f(16) = 16$ at $x = 16$
No global min.

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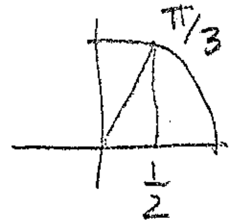
1. Find the global extrema of the function $f(x) = \sin(x) - \frac{x}{2}$ on the closed interval $\left[0, \frac{\pi}{2}\right]$.

$$f'(x) = \cos(x) - \frac{1}{2} = 0$$

$$\cos(x) = \frac{1}{2}$$



Critical point: $x = \frac{\pi}{3}$

$$f(0) = \sin(0) - \frac{0}{2} = 0 - 0 = 0$$

$$f\left(\frac{\pi}{3}\right) = \sin\left(\frac{\pi}{3}\right) - \frac{\pi/3}{2} = \frac{\sqrt{3}}{2} - \frac{\pi}{6} = \frac{3\sqrt{3} - \pi}{6} = \frac{3 \cdot 1.7 - 3.1}{6} = \frac{2}{6} = \frac{1}{3} = 0.3$$

$$f\left(\frac{\pi}{2}\right) = \sin\left(\frac{\pi}{2}\right) - \frac{\pi/2}{2} = 1 - \frac{\pi}{4} = \frac{4 - \pi}{4} \approx \frac{4 - 3.1}{4} = \frac{0.9}{4} \approx 0.22$$

f has a global min of $f(0) = 0$ at $x = 0$
f has a global max of $f\left(\frac{\pi}{3}\right) = \frac{3\sqrt{3} - \pi}{6}$ at $x = \frac{\pi}{3}$

2. Find the global extrema of the function $f(x) = x - 2\sqrt{x}$ on the open interval $(0, 9)$.

$$f(x) = x - 2x^{1/2}$$

$$f'(x) = 1 - 2 \cdot \frac{1}{2} x^{-1/2} = 1 - \frac{1}{\sqrt{x}} = 0$$

$$1 = \frac{1}{\sqrt{x}}$$

$$\sqrt{x} = 1$$

$$x = 1$$

Only one critical point!

critical point

$$f''(x) = \frac{1}{2} x^{-3/2} = \frac{1}{2\sqrt{x}^3}$$

$$f''(x) = \frac{1}{2\sqrt{1}} = \frac{1}{2} > 0,$$ so local min. at $x = 1$

f has a global min of $f(1) = -1$ at $x = 1$
No global min.