Name: ______________________________

Quiz 16

MATH 200
March 29, 2022

1. (10 points) Use the second derivative test to find the local extrema of $f(x) = x^3 - 2x^2 + x$.

   To find the critical points, look at $f'(x) = 3x^2 - 4x + 1 = (3x-1)(x-1) = 0$.
   We can now read off the critical points as $x = 1/3$ and $x = 1$.

   The second derivative is $f''(x) = 6x - 4$.

   At critical point $x{=}\frac{1}{3}$: $f''\left(\frac{1}{3}\right) = 6{\cdot}\frac{1}{3}{-}4 = -2 < 0$ $\boxed{f \text{ has local max of } f\left(\frac{1}{3}\right) = \frac{1}{27}{-}\frac{2}{9}{+}\frac{1}{3}{=}\frac{4}{27} \quad \text{at} \quad x{=}\frac{1}{3}}$

   At critical point $x{=}1$: $f''(1) = 6{\cdot}1{-}4 = 2 > 0$ $\boxed{f \text{ has local min of } f(1) = 1^3{-}2{\cdot}1^2{+}1{+}1 = 1 \text{ at } x{=}1}$

2. (10 points) This problem concerns the function $f(x) = xe^x$

   (a) Find the intervals on which f is increasing/decreasing.

   $f'(x) = e^x + xe^x = e^x(1+x)$ so critical point is $x = -1$.
   Notice $f'(x) > 0$ whenever $x > -1$, so $\boxed{f(x) \text{ increases on } (-1, \infty).}$
   Notice $f'(x) < 0$ whenever $x < -1$, so $\boxed{f(x) \text{ decreases on } (-\infty, -1).}$

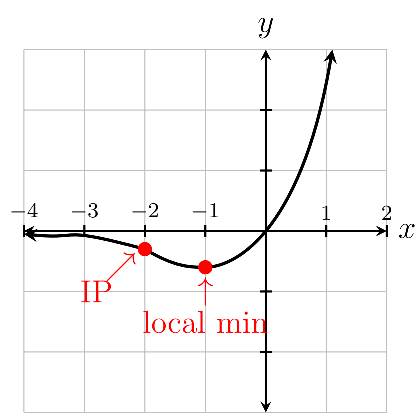
   (b) Find the intervals on which f is concave up/down.

   $f''(x) = e^x + e^x + xe^x = e^x(1+1+x) = e^x(2+x)$, which equals 0 at $x = -2$.
   Notice $f''(x) > 0$ whenever $x > -2$, so $\boxed{f(x) \text{ concave up on } (-2, \infty).}$
   Notice $f''(x) < 0$ whenever $x < -2$, so $\boxed{f(x) \text{ concave down on } (-\infty, -2).}$

   (c) List any inflection points.

   Concavity changes from down to up at $x{=}-2$,
   so inflection point is $(-2, f(-2)) = (-2, -2e^{-2})$
   $= \boxed{\left(-2, -\frac{2}{e^2}\right)}$

   (d) Based on this information, sketch the graph of f.
   Note: $f(0) = 0e^0 = 0$
   Also local min of $f(-1) = -1{\cdot}e^{-1} = -\frac{1}{e}$ at $x = -1$
   Inflection point: $(-2, f(-2)) = (-2, -2e^{-2}) = \left(-2, -\frac{2}{e^2}\right)$



Name: ______________________________

QUIZ 16

MATH 200
March 29, 2022

1. (10 points) Use the second derivative test to find the local extrema of $f(x) = 2x^3 - 3x^2 + 10$.

To find the critical points, look at $f'(x) = 6x^2 - 6x = 6x(x-1) = 0$.
We can now read off the critical points as $x = 0$ and $x = 1$.

The second derivative is $f''(x) = 12x - 6$.

At critical point x=0: $f''(0) = 12{\cdot}0 - 6 = -6 < 0$ $\boxed{f \text{ has local max of } f(0) = 10 \text{ at } x{=}0}$

At critical point x=1: $f''(1) = 12{\cdot}1 - 6 = 6 > 0$ $\boxed{f \text{ has local min of } f(1) = 9 \text{ at } x{=}1}$

2. (10 points) This problem concerns the function $f(x) = xe^x$

(a) Find the intervals on which f is increasing/decreasing.

$f'(x) = e^x + xe^x = e^x(1+x)$ so critical point is $x = -1$.
Notice $f'(x) > 0$ whenever $x > -1$, so $\boxed{f(x) \text{ increases on } (-1, \infty).}$
Notice $f'(x) < 0$ whenever $x < -1$, so $\boxed{f(x) \text{ decreases on } (-\infty, -1).}$

(b) Find the intervals on which f is concave up/down.

$f''(x) = e^x + e^x + xe^x = e^x(1+1+x) = e^x(2+x)$, which equals 0 for $x = -2$.
Notice $f''(x) > 0$ whenever $x > -2$, so $\boxed{f(x) \text{ concave up on } (-2, \infty).}$
Notice $f''(x) < 0$ whenever $x < -2$, so $\boxed{f(x) \text{ concave down on } (-\infty, -2).}$

(c) List any inflection points.

Concavity changes from down to up at $x = -2$,
so inflection point is $(-2, f(-2)) = (-2, -2e^{-2})$
$= \boxed{\left(-2, -\frac{2}{e^2}\right)}$

(d) Based on this information, sketch the graph of f.
Note: $f(0) = 0e^0 = 0$
Also local min of $f(-1) = -1{\cdot}e^{-1} = -\frac{1}{e}$ at $x = -1$
Inflection point: $(-2, f(-2)) = (-2, -2e^{-2}) = \left(-2, -\frac{2}{e^2}\right)$