Name: ______________________________

QUIZ 15

MATH 200
March 24, 2022

1. (10 points) This problem concerns the function $f(x) = \dfrac{x}{1+x^2}$.

   (a) Find the intervals on which f increases and on which it decreases.

$$\begin{aligned} f'(x) &= \frac{1\cdot(1+x^2) - x(0+2x)}{(1+x^2)^2} \\ &= \frac{1-x^2}{(1+x^2)^2} \\ &= \frac{(1-x)(1+x)}{(1+x^2)^2} \end{aligned}$$

$\longleftarrow$ critical points are 1 and -1

	$(-\infty,-1)$	$(-1,1)$	$(1,\infty)$
$(1+x^2)^2$	+ + + + + + + + + +	+ + + + + + + + + +	+ + + + + + + + + + + +
$(1+x)$	− − − − − − − − − −	+ + + + + + + + + +	+ + + + + + + + + + + +
$(1-x)$	+ + + + + + + + + +	+ + + + + + + + + +	− − − − − − − − − − − −
$f'(x) = (x+1)(x-1)/(1+x^2)^2$	− − − − − − − − − −	+ + + + + + + + + +	− − − − − − − − − − − −

Answer: f increases on $(-1, 1)$, and decreases on $(-\infty, -1) \cup (1, \infty)$

   (b) Use your answer from part (a) to identify the locations (x values) of any local extrema of f.

By the first derivative test, f has a local minimum at $x = -1$ and a local maximum at $x = 1$

2. (10 points) The graph of the **derivative** $f'(x)$ of a function f is shown below.

   (a) State the critical points of f.

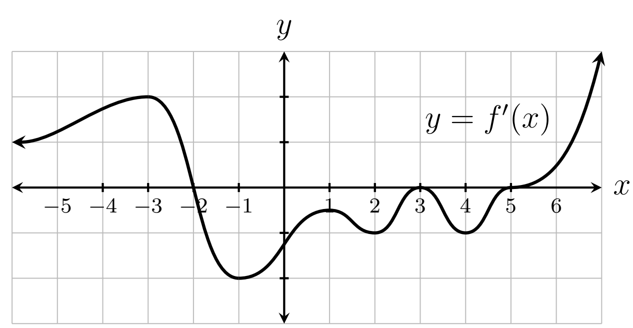
   -2, 3 and 5 since $f'(x)$=0 for these x values.

   (b) State the interval(s) on which f increases.

   On $(-\infty, -2)$ & $(5, \infty)$ since $f'(x)$>0 there.

   (c) State the interval(s) on which f decreases.

   On $(-2, 3)$ & $(3, 5)$ since $f'(x)$<0 there



   (d) Does f have a local maximum? Where?

   Yes, at $x = -2$ as $f'(x)$ goes + to − there.

   (e) Does f have a local minimum? Where?

   Yes, at $x = 5$ as $f'(x)$ goes − to + there.

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Quiz 15

MATH 200
March 24, 2022

1. (10 points) This problem concerns the function $f(x) = \ln\left(x^2e^x + 1\right)$.

   (a) Find the intervals on which f increases and on which it decreases.

$$f'(x) = \frac{2xe^x + x^2e^x \;\; + 0}{x^2e^x + 1} = \frac{e^x(2x + x^2)}{x^2e^x + 1} = \frac{e^x(2 + x)x}{x^2e^x + 1}$$

$\longleftarrow$ critical points are -2 and 0

(Because $e^x > 0$ and $x^2e^x + 1 > 0$ for all x, so the only way to get $f'(x) = 0$ is $x = -2$ or $x = 0$.)

	-2		0		
$++++++++++$		$+++++++++$		$++++++++++++$	e^x
$----------$		$+++++++++$		$++++++++++++$	$(2+x)$
$----------$		$---------$		$++++++++++++$	x
$++++++++++$		$+++++++++$		$++++++++++++$	$x^2e^x + 1$
$++++++++++$		$---------$		$++++++++++++$	$f'(x) = e^x(2+x)x/(x^2e^x + 1)$

Answer: f increases on $(-\infty, -2) \cup (0, \infty)$, and decreases on $(-2, 0)$

   (b) Use your answer from part (a) to identify the locations (x values) of any local extrema of f.

By the first derivative test, f has a local maximum at $x = -2$ and a local minimum at $x = 0$

2. (10 points) The graph of the **derivative** $f'(x)$ of a function f is shown below.

   (a) State the critical points of f.

   -4, -2 and 3 as $f'(x)=0$ for these x values.

   (b) State the interval(s) on which f increases.

   On $(-4, -2)$ & $(-2, 3)$ since $f'(x) > 0$ there.

   (c) State the interval(s) on which f decreases.

   On $(-\infty, -4)$ & $(3, \infty)$ since $f'(x) < 0$ there

   (d) Does f have a local maximum? Where?

   Yes, at $x = 3$ as $f'(x)$ goes $+$ to $-$ there.

   (e) Does f have a local minimum? Where?

   Yes, at $x = -4$ as $f'(x)$ goes $-$ to $+$ there.