Name: ______________________________ **Quiz 14** **MATH 200** March 15, 2022

Directions Use logarithmic differentiation to find the derivatives of the given functions.

1. $y = (5x+3)^x$

$$\begin{aligned}
\ln|y| &= \ln\left|(5x+3)^x\right| \\
\ln|y| &= x\ln\left|5x+3\right| \\
D_x\Big[\ln|y|\Big] &= D_x\Big[x\ln\left|5x+3\right|\Big] \\
\frac{y'}{y} &= 1\cdot\ln\left|5x+3\right| + x\cdot\frac{5}{5x+3} \\
y' &= y\left(\ln\left|5x+3\right| + \frac{5x}{5x+3}\right) \\
y' &= (5x+3)^x\left(\ln\left|5x+3\right| + \frac{5x}{5x+3}\right)
\end{aligned}$$

2. $y = \sqrt{x}\,\sin(x)\cos(x)$

$$\begin{aligned}
\ln|y| &= \ln\left|\sqrt{x}\,\sin(x)\cos(x)\right| \\
\ln|y| &= \ln\left|x^{1/2}\,\sin(x)\cos(x)\right| \\
\ln|y| &= \ln\left|x^{1/2}\right| + \ln|\sin(x)| + \ln|\cos(x)| \\
\ln|y| &= \frac{1}{2}\ln\left|x\right| + \ln|\sin(x)| + \ln|\cos(x)| \\
D_x\Big[\ln|y|\Big] &= D_x\Big[\frac{1}{2}\ln\left|x\right| + \ln|\sin(x)| + \ln|\cos(x)|\Big] \\
\frac{y'}{y} &= \frac{1}{2}\cdot\frac{1}{x} + \frac{\cos(x)}{\sin(x)} - \frac{\sin(x)}{\cos(x)} \\
y' &= y\left(\frac{1}{2x} + \cot(x) - \tan(x)\right) \\
y' &= \sqrt{x}\,\sin(x)\cos(x)\left(\frac{1}{2x} + \cot(x) - \tan(x)\right) \\
y' &= \frac{\sqrt{x}\,\sin(x)\cos(x)}{2x} + \sqrt{x}\cos^2(x) - \sqrt{x}\sin^2(x) \\
y' &= \frac{\sin(x)\cos(x)}{2\sqrt{x}} + \sqrt{x}\cos^2(x) - \sqrt{x}\sin^2(x)
\end{aligned}$$

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March 15, 2022

Directions Use logarithmic differentiation to find the derivatives of the given functions

1. $y = x^{5x+3}$

$$\begin{aligned}
\ln|y| &= \ln\left|x^{5x+3}\right| \\
\ln|y| &= (5x+3)\ln|x| \\
D_x\Big[\ln|y|\Big] &= D_x\Big[(5x+3)\ln|x|\Big] \\
\frac{y'}{y} &= 5\cdot\ln|x| + (5x+3)\cdot\frac{1}{x} \\
y' &= y\left(5\ln|x| + \frac{5x+3}{x}\right) \\
y' &= x^{5x+3}\left(5\ln|x| + \frac{5x+3}{x}\right)
\end{aligned}$$

2. $y = x^2\cos(x)\sin(x)$

$$\begin{aligned}
\ln|y| &= \ln\left|x^2\cos(x)\sin(x)\right| \\
\ln|y| &= \ln\left|x^2\right| + \ln|\cos(x)| + \ln|\sin(x)| \\
\ln|y| &= 2\ln|x| + \ln|\cos(x)| + \ln|\sin(x)| \\
D_x\Big[\ln|y|\Big] &= D_x\Big[2\ln|x| + \ln|\cos(x)| + \ln|\sin(x)|\Big] \\
\frac{y'}{y} &= 2\cdot\frac{1}{x} - \frac{\sin(x)}{\cos(x)} + \frac{\cos(x)}{\sin(x)} \\
y' &= y\left(\frac{2}{x} - \tan(x) + \cot(x)\right) \\
y' &= x^2\cos(x)\sin(x)\left(\frac{2}{x} - \tan(x) + \cot(x)\right) \\
y' &= 2x\cos(x)\sin(x) - x^2\sin^2(x) + x^2\cos^2(x)
\end{aligned}$$