Name: ______________________________

QUIZ 13

MATH 200
March 3, 2022

1. This problem concerns the equation $x^2 + xy + y^2 = 7$

(a) Find $\dfrac{dy}{dx}$.

$$\begin{aligned} D_x\Big[x^2 + xy + y^2\Big] &= D_x\Big[7\Big] \\ 2x + 1 \cdot y + xy' + 2yy' &= 0 \\ xy' + 2yy' &= -y - 2x \\ y'(x + 2y) &= -y - 2x \\ y' &= \frac{-y - 2x}{x + 2y} \end{aligned}$$

Therefore $\boxed{\dfrac{dy}{dx} = \dfrac{-y - 2x}{x + 2y}}$

(b) Use your answer from part (a) to find the slope of the tangent to the graph of $x^2 + xy + y^2 = 7$ at the point $\big(2, -3\big)$.

$$\left.\frac{dy}{dx}\right|_{(x,y)=(2,-3)} = \frac{-1\cdot(-3) - 2\cdot 2}{2 + 2\cdot(-3)} = \boxed{\frac{1}{4}}$$

Name: ______________________________ **Quiz 13** **MATH 200** March 3, 2022

1. This problem concerns the equation $xy^3 = xy + 6$

   (a) Find $\dfrac{dy}{dx}$.

$$\begin{aligned} D_x\left[xy^3\right] &= D_x\left[xy+6\right] \\ 1\cdot y^3 + x\cdot 3y^2y' &= 1\cdot y + xy' + 0 \\ y^3 + 3xy^2y' &= y + xy' \\ 3xy^2y' - xy' &= y - y^3 \\ y'(3xy^2 - x) &= y - y^3 \\ y' &= \frac{y-y^3}{3xy^2 - x} \end{aligned}$$

Therefore $\boxed{\dfrac{dy}{dx} = \dfrac{y-y^3}{3xy^2-x}}$

   (b) Use your answer from part (a) to find the slope of the tangent to the graph of $xy^3 = xy + 6$ at the point $\big(1, 2\big)$.

$$\left.\frac{dy}{dx}\right|_{(x,y)=(1,2)} = \frac{2-2^3}{3\cdot 1\cdot 2^2 - 1} = \boxed{-\frac{6}{11}}$$