Name: ____________________ QUIZ 12 MATH 200

March 1, 2022

1. Differentiate: $\sec^{-1}(x)\ln(x)$

$$D_x\Big[\sec^{-1}(x)\ln(x)\Big] = \frac{1}{|x|\sqrt{x^2-1}}\ln(x) + \sec^{-1}(x)\frac{1}{x} = \boxed{\frac{\ln(x)}{|x|\sqrt{x^2-1}} + \frac{\sec^{-1}(x)}{x}} \quad \text{(product rule)}$$

2. Differentiate: $\sin^{-1}\big(x\ln(x)\big)$

$$D_x\Big[\sin^{-1}\big(x\ln(x)\big)\Big] = \frac{1}{\sqrt{1-(x\ln(x))^2}}\cdot D_x\Big[x\ln(x)\Big] = \frac{1}{\sqrt{1-(x\ln(x))^2}}\cdot\left(1\cdot\ln(x)+x\frac{1}{x}\right)$$

$$= \boxed{\frac{\ln(x)+1}{\sqrt{1-(x\ln(x))^2}}}$$

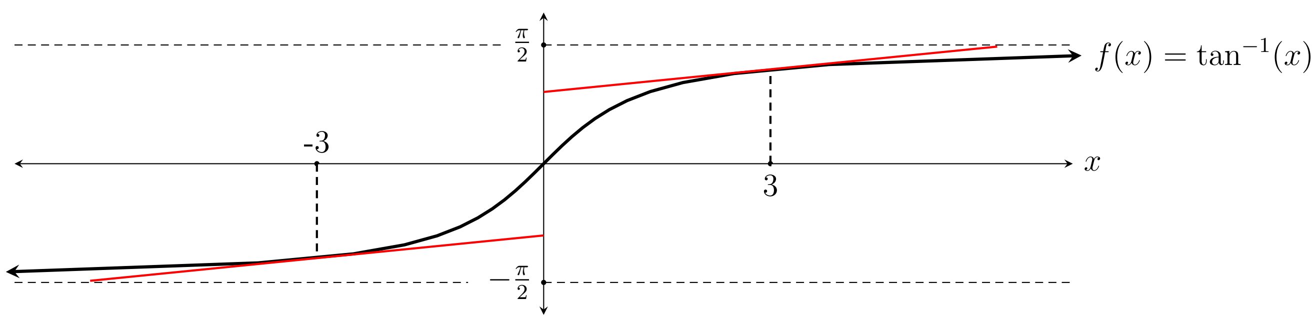
3. Find all x for which the tangent line to $f(x)=\tan^{-1}(x)$ at $\big(x, f(x)\big)$ has slope $m=\frac{1}{10}$.

We just need to solve the equation $f'(x)=\frac{1}{10}$, which is

$$\begin{aligned} \frac{1}{1+x^2} &= \frac{1}{10} \\ 1+x^2 &= 10 \\ x^2 &= 9 \\ x &= \pm 3. \end{aligned}$$

Answer: The slope of the tangent to $y=\tan^{-1}(x)$ is $1/10$ at $x=3$ and $x=-3$.

Notice that this seems entirely reasonable when we look at the graph:



4. An object moving on a line is $s(t)=t^3-3t^2$ meters from its starting point at time t seconds. Find the object's acceleration when its velocity is -3 meters per second.

The velocity at time t is $s'(t)=3t^2-6t$. Its acceleration at time t is $a(t)=s''(t)=6t-6$ To find at which time t the velocity is -3 meters per second, we solve $s'(t)=-3$, which is:

$$\begin{aligned} 3t^2-6t &= -3 \\ 3t^2-6t+3 &= 0 \\ 3(t^2-2t+1) &= 0 \\ 3(t-1)(t-1) &= 0 \end{aligned}$$

So the velocity is -3 meters per second at time $t=1$ second. At this time the acceleration is $a(1)=6\cdot 1-6=$ $\boxed{0 \text{ meters per second per second}}$

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QUIZ 12

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1. Differentiate: $x^5 \tan^{-1}(x)$

$$D_x\Big[x^5 \tan^{-1}(x)\Big] = 5x^4 \tan^{-1}(x) + x^5 \frac{1}{1+x^2} = \boxed{5x^4 \tan^{-1}(x) + \frac{x^5}{1+x^2}} \qquad \text{(product rule)}$$

2. Differentiate: $\sec^{-1}\big(x\ln(x)\big)$

$$D_x\Big[\sec^{-1}\big(x\ln(x)\big)\Big] = \frac{1}{\big|x\ln(x)\big|\sqrt{\big(x\ln(x)\big)^2-1}} \cdot D_x\Big[x\ln(x)\Big]$$

$$= \frac{1}{\big|x\ln(x)\big|\sqrt{\big(x\ln(x)\big)^2-1}} \cdot \left(1\cdot\ln(x) + x\frac{1}{x}\right) = \boxed{\frac{\ln(x)+1}{\big|x\ln(x)\big|\sqrt{\big(x\ln(x)\big)^2-1}}}$$

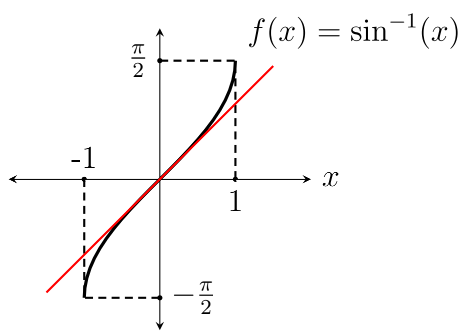
3. Find all x for which the tangent line to $f(x) = \sin^{-1}(x)$ at $\big(x, f(x)\big)$ has slope $m = 1$.

We just need to solve the equation $f'(x) = 1$, which is

$$\begin{aligned} \frac{1}{\sqrt{1-x^2}} &= 1 \\ 1 &= \sqrt{1-x^2} \\ 1^2 &= \sqrt{1-x^2}^2 \\ 1 &= 1-x^2 \\ x^2 &= 0 \\ x &= 0 \end{aligned}$$

Answer: The slope of the tangent to $y = \sin^{-1}(x)$ is 1 at $x = 0$.

This is reasonable when we look at the graph. The tangent at x=0 indeed appears to have slope 1:



4. An object moving on a line is $s(t) = t^3 - 3t^2$ meters from its starting point at time t seconds. Find the object's velocity at the instant its acceleration is 6 meters per second per second.

The velocity at time t is $v(t) = s'(t) = 3t^2 - 6t$. Its acceleration at time t is $a(t) = v'(t) = 6t - 6$. To find the time t when acceleration is 6 meters per second per second, we solve $a(t) = 6$, which is:

$$\begin{aligned} 6t - 6 &= 6 \\ 6t &= 12 \\ t &= 2 \end{aligned}$$

So the acceleration is 6 meters per second per second at time $t = 2$ seconds. At this time the velocity is $v(2) = 3\cdot 2^2 - 6\cdot 2$ $\boxed{\text{0 meters per second}}$