

1. Use the fundamental theorem of calculus to find the definite integrals.

$$(a) \int_1^e \frac{1}{x} dx = \left[\ln|x| \right]_1^e = \ln|e| - \ln|1| = 1 - 0 = \boxed{1}$$

$$(b) \int_{-1}^1 (x^2 + x) dx = \left[\frac{x^3}{3} + \frac{x^2}{2} \right]_{-1}^1 = \left(\frac{1^3}{3} + \frac{1^2}{2} \right) - \left(\frac{(-1)^3}{3} + \frac{(-1)^2}{2} \right)$$

$$= \left(\frac{1}{3} + \frac{1}{2} \right) - \left(-\frac{1}{3} + \frac{1}{2} \right)$$

$$= \frac{1}{3} + \frac{1}{2} + \frac{1}{3} - \frac{1}{2} = \boxed{\frac{2}{3}}$$

$$(c) \int_0^1 \frac{1}{1+x^2} dx = \left[\tan^{-1}(x) \right]_0^1 = \tan^{-1}(1) - \tan^{-1}(0)$$

$$= \frac{\pi}{4} - 0 = \boxed{\frac{\pi}{4}}$$

2. Find $\int \cos^6(x) \sin(x) dx = \int (\cos(x))^6 \sin(x) dx$

$$\left. \begin{array}{l} \text{Let } u = \cos(x) \\ \text{so } \frac{du}{dx} = -\sin(x) \\ \text{and } du = -\sin(x) dx \\ \text{so } -du = \sin(x) dx \end{array} \right\} = \int u^6 (-du) = -\int u^6 du$$

$$= -\frac{u^7}{7} + C$$

$$= \boxed{-\frac{\cos^7(x)}{7} + C}$$