Name: ______________________________

Quiz 11 | **MATH 200**
February 24, 2022

1. Differentiate: $\sec(x)\ln(x)$ $\quad D_x\Big[\sec(x)\ln(x)\Big] = \boxed{\sec(x)\tan(x)\ln(x) + \sec(x)\frac{1}{x}}$ (product rule)

$$= \boxed{\sec(x)\left(\tan(x)\ln(x) + \frac{1}{x}\right)}$$

2. Differentiate: $\sec\big(\ln(x)\big)$ $\quad D_x\Big[\sec\big(\ln(x)\big)\Big] = \boxed{\sec\big(\ln(x)\big)\tan\big(\ln(x)\big)\frac{1}{x}}$ (chain rule)

$$= \boxed{\frac{\sec\big(\ln(x)\big)\tan\big(\ln(x)\big)}{x}}$$

3. Differentiate: $\ln\big(\sec(x)\big)$ $\quad D_x\Big[\ln\big(\sec(x)\big)\Big] = \frac{\sec(x)\tan(x)}{\sec(x)} = \boxed{\tan(x)}$ (chain rule)

4. Differentiate: $4x + \frac{xe^x}{\ln(x)}$

$D_x\left[4x + \frac{xe^x}{\ln(x)}\right] = D_x\Big[4x\Big] + D_x\left[\frac{xe^x}{\ln(x)}\right]$ (sum/diff rule)

$= 4 + \frac{D_x\left[xe^x\right]\ln(x) - xe^x D_x\left[\ln(x)\right]}{\big(\ln(x)\big)^2}$ (power rule and quotient rule)

$= 4 + \frac{(1\cdot e^x + xe^x)\ln(x) - xe^x\frac{1}{x}}{\big(\ln(x)\big)^2}$ (product rule, etc.)

$= \boxed{4 + \frac{e^x\Big((1+x)\ln(x) - 1\Big)}{\big(\ln(x)\big)^2}}$ (simplify)

5. Find all x for which the tangent line to $f(x) = \ln\left|x^3 - 6x^2 - 15x\right|$ at $\big(x, f(x)\big)$ has slope 0.

We need to solve the equation $f'(x) = 0$, which is

$$\frac{3x^2 - 12x - 15}{x^3 - 6x^2 - 15x} = 0$$

$$\begin{aligned} 3x^2 - 12x - 15 &= 0 \quad \text{(cross multiply)} \\ 3\left(x^2 - 4x - 5\right) &= 0 \\ 3(x+1)(x-5) &= 0 \end{aligned}$$

Answer: Tangent slope is zero at $x = -1$ and $x = 5$.

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1. Differentiate: $\tan\big(\ln(x)\big)$ $\quad D_x\Big[\tan\big(\ln(x)\big)\Big] = \sec^2\big(\ln(x)\big)\dfrac{1}{x} = \boxed{\dfrac{\sec^2\big(\ln(x)\big)}{x}}$ (chain rule)

2. Differentiate: $\ln(x)\tan(x)$ $\quad D_x\Big[\ln(x)\tan(x)\Big] = \boxed{\dfrac{1}{x}\tan(x) + \ln(x)\sec^2(x)}$ (product rule)

3. Differentiate: $\ln\big(\tan(x)\big)$ $\quad D_x\Big[\ln\big(\tan(x)\big)\Big] = \boxed{\dfrac{\sec^2(x)}{\tan(x)}}$ (chain rule)

4. Differentiate: $4 + \dfrac{x\ln(x)}{e^x}$

$$D_x\left[4 + \frac{x\ln(x)}{e^x}\right] = D_x\Big[4\Big] + D_x\left[\frac{x\ln(x)}{e^x}\right] \quad \text{(sum/diff rule)}$$

$$= 0 + \frac{D_x\big[x\ln(x)\big]e^x - x\ln(x)D_x\big[e^x\big]}{\big(e^x\big)^2} \quad \text{(quotient rule)}$$

$$= \frac{\left(1\cdot\ln(x) + x\dfrac{1}{x}\right)e^x - x\ln(x)e^x}{\big(e^x\big)^2} \quad \text{(product rule, etc.)}$$

$$= \frac{e^x\Big(\ln(x) + 1 - x\ln(x)\Big)}{\big(e^x\big)^2} = \boxed{\frac{\ln(x) + 1 - x\ln(x)}{e^x}} \quad \text{(simplify)}$$

5. Find all x for which the tangent line to $f(x) = \ln\left|x^3 - 9x^2 + 24x\right|$ at $\big(x, f(x)\big)$ has slope 0.
We need to solve the equation $f'(x) = 0$, which is

$$\frac{3x^2 - 18x + 24}{x^3 - 9x^2 + 24x} = 0$$

$$\begin{aligned} 3x^2 - 18x + 24 &= 0 \quad \text{(cross multiply)} \\ 3\big(x^2 - 6x + 8\big) &= 0 \\ 3(x-2)(x-4) &= 0 \end{aligned}$$

Answer: Tangent slope is zero at $x = 2$ and $x = 4$.