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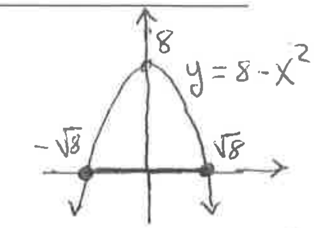
MATH 200 – QUIZ 11

April 2, 2015

1. Consider the function $g(x) = x\sqrt{8-x^2}$. $= x\,(8-x^2)^{1/2}$



(a) Find all critical points of $g(x)$. First, notice that for x to be in the domain of $g(x)$ we must have $8 - x^2 \geq 0$, meaning $-\sqrt{8} \leq x \leq \sqrt{8}$, so the domain of $g(x)$ is the interval $[-\sqrt{8}, \sqrt{8}] = [-2\sqrt{2}, 2\sqrt{2}]$. All crit. pts. must be in $[-\sqrt{8}, \sqrt{8}]$

Find critical points: $g'(x) = (1)\sqrt{8-x^2} + x\,\frac{1}{2}(8-x^2)^{-1/2}(-2x)$

$$= \sqrt{8-x^2} - \frac{x^2}{\sqrt{8-x^2}}$$

$$= \frac{\sqrt{8-x^2}\sqrt{8-x^2} - x^2}{\sqrt{8-x^2}} = \frac{8 - x^2 - x^2}{\sqrt{8-x^2}}$$

$$= \frac{8-2x^2}{\sqrt{8-x^2}} = \frac{2(4-x^2)}{\sqrt{8-x^2}} = \boxed{\frac{2(2-x)(2+x)}{\sqrt{8-x^2}}}$$

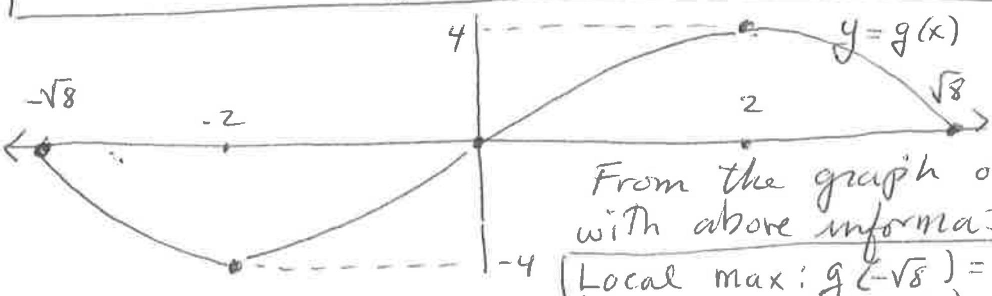
From this we see that $g'(x)$ is undefined if $x = \pm\sqrt{8} = \pm 2\sqrt{2}$ and $g'(x) = 0$ if $x = \pm 2$. $\boxed{\text{Critical points: } 2, -2, \sqrt{8}, -\sqrt{8}}$

(b) Find the locations of the local minima and maxima (if any) of $g(x)$.

$-\sqrt{8}$ —— -2 —— 0 —— 2 —— $\sqrt{8}$

$[-\sqrt{8}, -2]$	$[-2, 2]$	$[2, \sqrt{8}]$	
+ + + +	+ + + + + + +	− − −	$2(2-x)$
− − − −	+ + + + + + +	+ + +	$(2+x)$
+ + + +	+ + + + + + +	+ + +	$\sqrt{8-x^2}$
− − − −	+ + + + + +	− − −	$g'(x) = \frac{2(2-x)(2+x)}{\sqrt{8-x^2}}$

$$\boxed{\begin{array}{l} \text{Local min: } g(-2) = -2\sqrt{8-(-2)^2} = -4 \text{ at } x = -2 \\ \text{Local max: } g(2) = 2\sqrt{8-2^2} = 4 \text{ at } x = 2 \end{array}}$$



From the graph of $y = g(x)$ sketched with above information, can also see:

$$\boxed{\begin{array}{l} \text{Local max: } g(-\sqrt{8}) = 0 \text{ at } -\sqrt{8} \\ \text{Local min } g(\sqrt{8}) = 0 \text{ at } \sqrt{8} \end{array}}$$