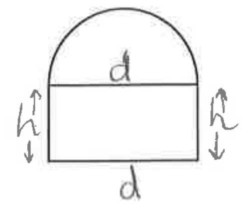
Name: Alexi

MATH 200 – QUIZ 11

Instructions: Show work and put a $\boxed{\text{box}}$ around your final answer.

April 4, 2013

1. You are designing a window consisting of a rectangle with a half-circle on top, as illustrated. The client can only afford 1 meter of window framing material. The framing material runs around the outside of the window and between the rectangular and semicircular regions. What should the diameter of the half-circle be to maximize the area of the window?



(a) Label the diagram with the appropriate variables. Find the function to be optimized.

Constraint - framing material

$$1 = \underbrace{h+d+h+d}_{\text{around rectangle}} + \underbrace{\frac{\pi d}{2}}_{\text{half circumference}}$$

$$1 = 2h + \left(2+\frac{\pi}{2}\right)d \quad \text{(relationship } h \leftrightarrow d\text{)}$$

$$\left(1-\left(2+\frac{\pi}{2}\right)d\right)\frac{1}{2} = h$$

$$h = \frac{1}{2} - \left(1+\frac{\pi}{4}\right)d$$

Function to be optimized

$$A = \frac{1}{2}\pi r^2 + hd$$

notice $d = 2r$ by definition of circle's radius and diameter

$$A = \frac{1}{2}\pi\left(\frac{d}{2}\right)^2 + hd \quad \text{(two vars)}$$

$$A(d) = \frac{1}{2}\pi\left(\frac{d^2}{4}\right) + \left(\frac{1}{2} - \left(1+\frac{\pi}{4}\right)d\right)d$$

$$A(d) = \frac{\pi}{8}d^2 + \frac{1}{2}d - \left(1+\frac{\pi}{4}\right)d^2$$

(single-variable)

can consolidate some more if you want

(b) Find the critical points of this function.

$$A'(d) = \frac{\pi}{4}d + \frac{1}{2} - \left(2+\frac{\pi}{2}\right)d \quad \text{(never undef)}$$

$$0 = \frac{\pi}{4}d + \frac{1}{2} - \left(2+\frac{\pi}{2}\right)d \quad (A'=0)$$

$$-\frac{1}{2} = \frac{\pi}{4}d - \left(2+\frac{\pi}{2}\right)d$$

$$-\frac{1}{2} = d\left(\frac{\pi}{4} - 2 - \frac{\pi}{2}\right) = d\left(-2-\frac{\pi}{4}\right) \rightarrow \frac{-\frac{1}{2}}{\left(-2-\frac{\pi}{4}\right)} = d = \frac{1}{4+\frac{\pi}{2}}$$

(c) Use the first or second derivative test on the critical points that make sense in the context of this problem.

$$A''(d) = \frac{\pi}{4} + \frac{1}{2} - 2 - \frac{\pi}{2} = \frac{-3}{2} - \frac{\pi}{4} < 0 \text{ for all } d$$

in particular, $A''(\text{critical pt}) < 0$

so by the second derivative test, $d = \frac{1}{4+\frac{\pi}{2}}$ is the location of a maximum.

(d) Answer the question.

The diameter should be $\boxed{\frac{1}{4+\frac{\pi}{2}} \text{ meters}}$.

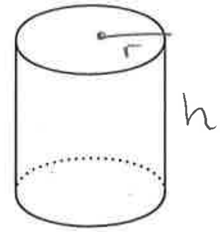
Name: Alexi

MATH 200 – QUIZ 11 1

Instructions: Show work and put a $\boxed{\text{box}}$ around your final answer.

April 4, 2013

1. You are designing a cylindrical can which has a bottom but no lid. The can must have a volume of 1000cm^3. What should the height and radius of the can be to minimize its surface area?



(a) Label the diagram with the appropriate variables. Find the function to be optimized.

Constraint – volume

$V = 1000 = \pi r^2 h$

$\frac{1000}{\pi r^2} = h$

(relationship between h and r)

Function to be optimized

$A = \underbrace{\pi r^2}_{\text{bottom}} + \underbrace{2\pi r h}_{\text{side of can}}$ (two variables!)

$A(r) = \pi r^2 + 2\cancel{\pi}\cancel{r}\left(\frac{1000}{\cancel{\pi} r^{\cancel{2}}}\right)$

$A(r) = \pi r^2 + \frac{2000}{r}$

single variable function

(b) Find the critical points of this function.

$A'(r) = 2\pi r - \frac{2000}{r^2}$ undef at $r=0$ (but r not in domain of A, so its not a crit. point)

$0 = 2\pi r - \frac{2000}{r^2}$ $(A' = 0)$

$2\pi r = \frac{2000}{r^2}$

$2\pi r^3 = 2000$ so $r = \sqrt[3]{\frac{1000}{\pi}} = \frac{10}{\sqrt[3]{\pi}}$ (critical point)

(c) Use the first or second derivative test on the critical points that make sense in the context of this problem.

We don't want a can with radius = 0, so ignore r = 0 critical point

$A''(r) = 2\pi + \frac{4000}{r^3} > 0$ for all positive r

in particular, A''(critical point) > 0 so by the second derivative test, $r = \frac{10}{\sqrt[3]{\pi}}$ is the location of a minimum of A(r)

(d) Answer the question.

$h = \frac{1000}{\pi r^2} = \frac{1000}{\pi}\frac{\pi^{-2/3}}{100} = \boxed{10\pi^{-5/3}\text{ cm}}$ and $\boxed{r = \frac{10}{\sqrt[3]{\pi}}\text{ cm}}$

plug in r

h

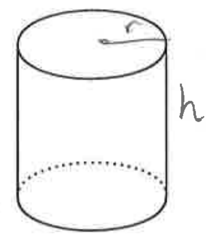
Name: Alexi

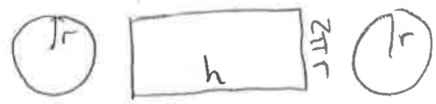
MATH 200 – QUIZ 11

Instructions: Show work and put a $\boxed{\text{box}}$ around your final answer.

April 4, 2013

1. You are designing a cylindrical can (with both a top and a bottom) that must have a volume of 1000cm^3. What should the height and radius of the can be to minimize its surface area?





(a) Label the diagram with the appropriate variables. Find the function to be optimized.

Constraint – volume

$V = 1000 = h \cdot \pi r^2$

$\dfrac{1000}{\pi r^2} = h$

(relationship between h and r)

Function to be optimized

$A = \underbrace{\pi r^2 + \pi r^2}_{\text{area of top \& bottom}} + \underbrace{2\pi r h}_{\text{area of rectangle}}$

$A = 2\pi r^2 + 2\pi r h$ (two vars!)

$A(r) = 2\pi r^2 + 2\pi r\left(\dfrac{1000}{\pi r^2}\right)$

$A(r) = 2\pi r^2 + \dfrac{2000}{r}$

single variable function

(b) Find the critical points of this function.

$A'(r) = 4\pi r - \dfrac{2000}{r^2}$ undefined at $r=0$ (but r not in domain of A, so not a critical point)

$0 = 4\pi r - \dfrac{2000}{r^2}$ $(A'=0)$

$\dfrac{2000}{r^2} = 4\pi r$

$2000 = 4\pi r^3$ so $r = \sqrt[3]{\dfrac{500}{\pi}}$ (critical point)

(c) Use the first or second derivative test on the critical points that make sense in the context of this problem.

We dont want a can with radius $=0$, so don't bother with $r=0$

$A''(r) = 4\pi + \dfrac{4000}{r^3} > 0$ for all positive radius

in particular, $A''(\text{critical point}) > 0$ so by the second derivative test, $r = \sqrt[3]{\dfrac{500}{\pi}}$ is the location of a minimum of $A(r)$

(d) Answer the question.

$h = \dfrac{1000}{\pi r^2} = \boxed{\dfrac{1000}{\pi}\left(\dfrac{500}{\pi}\right)^{-\frac{2}{3}} \text{ cm}}$ and $\boxed{r = \sqrt[3]{\dfrac{500}{\pi}} \text{ cm}}$

plug in r

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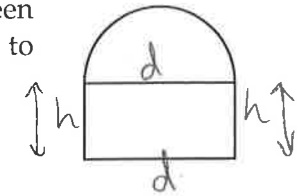
Instructions: Show work and put a $\boxed{\text{box}}$ around your final answer.

April 4, 2013

1. You are designing a window consisting of a rectangle with a half-circle on top, as illustrated. The client can only afford 1 meter of window framing material which will run along the very outside portion of the window; no framing material is required between the rectangular and semicircular regions. What should the diameter of the half-circle be to maximize the area of the window?



(a) Label the diagram with the appropriate variables. Find the function to be optimized.

Constraint – framing material

$$1 = \underbrace{h+d+h}_{\text{rectangle part}} + \underbrace{\frac{\pi d}{2}}_{\text{half-circle}}$$

$1 = 2h + (1+\frac{\pi}{2})d$ (relationship) $h \leftrightarrow d$

$(1-(1+\frac{\pi}{2})d)\frac{1}{2} = h$ (solving for h)

$h = \frac{1}{2} - (\frac{1}{2}+\frac{\pi}{4})d$

Function to be optimized

$$A = \underbrace{\tfrac{1}{2}\pi r^2}_{\text{half circle}} + \underbrace{hd}_{\text{rectangle}}$$

Remember radius is half of diameter

$A = \frac{1}{2}\pi\left(\frac{d}{2}\right)^2 + hd$ (two variables!)

$$A(d) = \frac{1}{2}\pi\frac{d^2}{4} + \underbrace{\left(\tfrac{1}{2} - (\tfrac{1}{2}+\tfrac{\pi}{4})d\right)}_{\text{from } h} d$$

$A(d) = \frac{\pi}{8}d^2 + \frac{1}{2}d - (\frac{1}{2}+\frac{\pi}{4})d^2$ can consolidate further if you want

single variable function

(b) Find the critical points of this function.

$A'(d) = \frac{\pi}{4}d + \frac{1}{2} - (1+\frac{\pi}{2})d$ (never undef)

$0 = \frac{\pi}{4}d + \frac{1}{2} - (1+\frac{\pi}{2})d$ $(A'=0)$

$-\frac{1}{2} = \frac{\pi}{4}d - d - \frac{\pi}{2}d = -d - \frac{\pi}{4}d = d(-1-\frac{\pi}{4})$

$\frac{-\frac{1}{2}}{(-1-\frac{\pi}{4})} = d = \frac{\frac{1}{2}}{1+\frac{\pi}{4}} = \frac{1}{2+\frac{\pi}{2}}$ so $d = \frac{1}{2+\frac{\pi}{2}}$ is the critical point of $A(d)$

(c) Use the first or second derivative test on the critical points that make sense in the context of this problem.

$A''(d) = \frac{\pi}{4} - (1+\frac{\pi}{2}) = -1-\frac{\pi}{4} < 0$ for all d

in particular, $A''(\text{critical point}) < 0$

so by second derivative test, $d = \frac{1}{2+\frac{\pi}{2}}$ is the location of a maximum of the area function.

(d) Answer the question.

The diameter should be $\boxed{\frac{1}{2+\frac{\pi}{2}} \text{ meters.}}$