Name: Richard

MATH 200 – QUIZ 10

I'm in the Thurs11 Thurs12 Thurs1 or Fri10 recitation. (Circle one)

November 7, 2012

1. This problem concerns the function $f(x) = (x-2)e^x$.

(a) Find the intervals on which $f(x)$ increases/decreases.

$$f'(x) = (1)e^x + (x-2)e^x$$
$$= e^x + xe^x - 2e^x$$
$$= xe^x - e^x$$
$$= (x-1)e^x$$

$x = 1$ is critical point

0 1

– – – – – – – – | + + $f'(x) = (x-1)e^x$

$$\boxed{\begin{array}{l} f(x) \text{ decreases on } (-\infty, 1) \\ f(x) \text{ increases on } (1, \infty) \end{array}}$$

Local minimum at $x=1$ is $f(1) = (1-2)e^1$
$= -e \approx -2.7$

(b) Find the intervals on which $f(x)$ is concave up/down.

$$f'(x) = (x-1)e^x$$
$$f''(x) = (1)e^x + (x-1)e^x$$
$$= e^x + xe^x - e^x$$
$$= xe^x$$

$x = 0$ is crit. pt for $f''(x)$

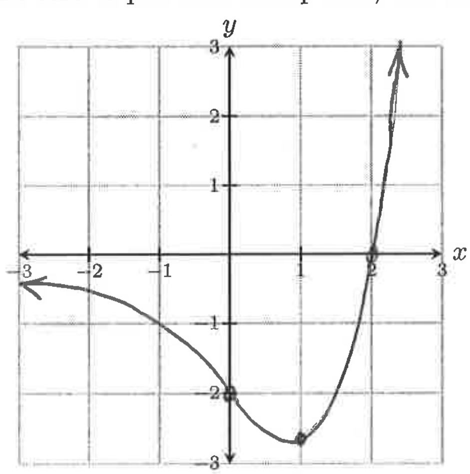
0

– – – – | + + + $f''(x) = xe^x$

$$\boxed{\begin{array}{l} f(x) \text{ concave down on } (-\infty, 0) \\ f(x) \text{ concave up on } (0, \infty) \end{array}}$$

Inflection point $(0, f(0)) = (0, (0-2)e^0)$
$= (0, -2)$

(c) Use the above information to sketch the graph of $f(x)$.
Be sure to plot inflection points, extrema and intercepts.



– – – – – – | – – | + + + $f'(x)$

– – – – – – | + + + + $f''(x)$

x-intercept $f(x) = 0$

$(x-2)e^x = 0$

$x = 2$

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November 7, 2012

1. This problem concerns the function $f(x) = 3x^{2/3} - 2x$. $= 3\sqrt[3]{x^2} - 2x$

(a) Find the intervals on which $f(x)$ increases/decreases.

$$f'(x) = 3 \cdot \frac{2}{3} x^{-1/3} - 2 = 2x^{-1/3} - 2 = 2\left(\frac{1}{\sqrt[3]{x}} - 1\right)$$

$0 \quad 1$

$- - - \mid + + + \mid - - - \quad f'(x)$

Note: $f'(0)$ is undefined.

$f'(1) = 0$

Critical points are 0 and 1.

$f(x)$ increases on $(0,1)$

$f(x)$ decreases on $(-\infty, 0)$ and $(1, \infty)$

(b) Find the intervals on which $f(x)$ is concave up/down.

$$f'(x) = 2x^{-1/3} - 2$$

$$f''(x) = -\frac{2}{3}x^{-4/3} - 0 = \frac{-2}{3x^{4/3}} = \frac{-2}{3\sqrt[3]{x^4}}$$

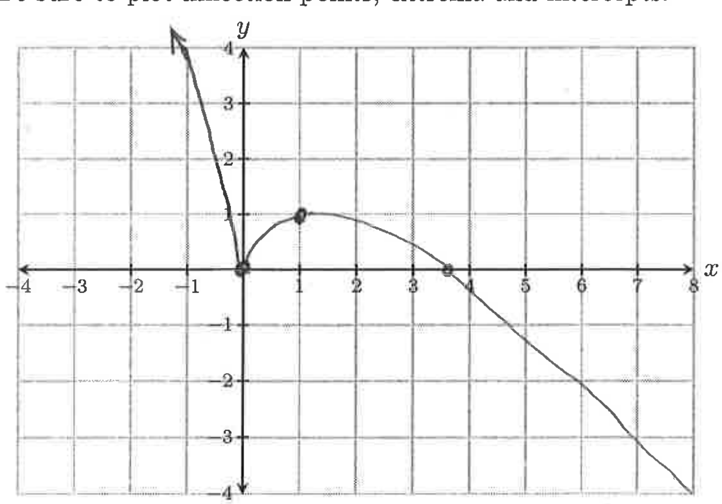
0

$- - - \mid - - - - \quad f''(x) = \frac{-2}{3\sqrt[3]{x^4}}$

Note: $f''(x)$ is always negative (not defined at $x = 0$)

$f(x)$ concave down on all of $\mathbb{R} = (-\infty, \infty)$.
No inflection points.

(c) Use the above information to sketch the graph of $f(x)$.
Be sure to plot inflection points, extrema and intercepts.



$- - - - - \mid + \mid - - - - - - - \quad f'(x)$

$- - - - - \mid - - - - - - - - - \quad f''(x)$

- Local min:
  $f(0) = 3\sqrt[3]{0^2} - 2 \cdot 0 = 0$
- Local max:
  $f(1) = 3\sqrt[3]{1^2} - 2 = 1$
- y-intercept: $f(0) = 0$
- x-intercept: $f(x) = 0$

$$3\sqrt[3]{x^2} - 2x = 0$$
$$3\sqrt[3]{x^2} = 2x$$
$$\left(3\sqrt[3]{x^2}\right)^3 = (2x)^3$$
$$27x^2 = 8x^3$$
$$x = \frac{27}{8}$$

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November 7, 2012

1. This problem concerns the function $f(x) = (2-x)e^x$.

(a) Find the intervals on which $f(x)$ increases/decreases.

$$f'(x) = (-1)e^x + (2-x)e^x$$
$$= -e^x + 2e^x - xe^x$$
$$= e^x - xe^x$$
$$= (1-x)e^x = 0$$

$x=1$ is crit. pt.

0 1

+ + | + + + | − − − $f'(x)$

$f(x)$ increasing on $(-\infty, 1)$
$f(x)$ decreasing on $(1, \infty)$

(b) Find the intervals on which $f(x)$ is concave up/down.

$$f'(x) = (1-x)e^x$$
$$f''(x) = (-1)e^x + (1-x)e^x$$
$$= -e^x + e^x - xe^x$$
$$= -xe^x$$

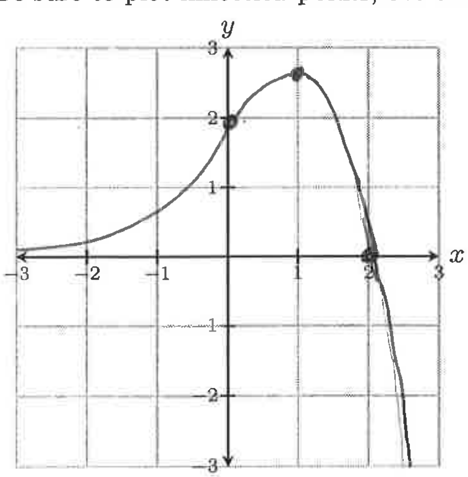
$x=0$ makes 2nd derivative 0

0

+ + + + | − − − − − − $f''(x) = -xe^x$

$f(x)$ concave up on $(-\infty, 0)$
$f(x)$ concave down on $(0, \infty)$

(c) Use the above information to sketch the graph of $f(x)$.
Be sure to plot inflection points, extrema and intercepts.



+ + + + + + + | − − − $f'(x)$

+ + + + + | − − − − − − − $f''(x)$

Local max at 1, $f(1) = (2-1)e^1 = e^1 = e \approx 2.7$

Inflection point, $(0, f(0)) = (0, (2-0)e^0) = (0, 2)$

x-intercept: $f(x) = 0$
$(2-x)e^x = 0$

$x = 2$

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1. This problem concerns the function $f(x) = 2x - 3x^{2/3}$. $= 2x - 3\sqrt[3]{x^2}$

(a) Find the intervals on which $f(x)$ increases/decreases.

$f'(x) = 2 - 3\cdot\frac{2}{3}x^{-1/3} = 2 - 2x^{-1/3}$

$= 2 - \frac{2}{\sqrt[3]{x}} = 2\left(1 - \frac{1}{\sqrt[3]{x}}\right)$

0 1

$+ + + + \mid - - - \mid + +$ $f'(x)$

$f'(0)$ is not defined, $f'(1) = 0$ } by inspection

Thus critical points are 0, 1.

$\boxed{\begin{array}{l} f(x) \text{ increases on } (-\infty, 0) \text{ and } (1, \infty) \\ f(x) \text{ decreases on } (0, 1) \end{array}}$

(b) Find the intervals on which $f(x)$ is concave up/down.

$f'(x) = 2 - 2x^{-1/3}$

$f''(x) = 0 + \frac{2}{3}x^{-4/3}$

$= \frac{2}{3x^{4/3}} = \frac{2}{3\sqrt[3]{x}^4}$

Note $f''(x)$ is always positive!

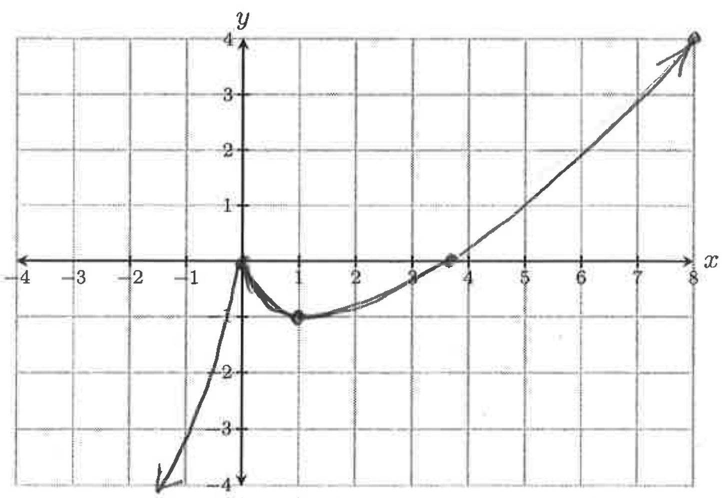
0

$+ + + \mid + + + + +$ $f''(x)$

$\boxed{f(x) \text{ concave up on all of } \mathbb{R} = (-\infty, \infty)}$

no inflection point.

(c) Use the above information to sketch the graph of $f(x)$.
Be sure to plot inflection points, extrema and intercepts.



$+ + + + \mid - \mid + + + + + + +$ $f'(x)$

$+ + + + + + + + + + +$ $f''(x)$

Local max:
$f(0) = 2\cdot 0 - 3\sqrt[3]{0^2} = 0$

Local min
$f(1) = 2\cdot 1 - 3\sqrt[3]{1^2} = 2 - 3 = -1$

y-intercept $f(0) = 0$

x-intercept $f(x) = 0$

$2x - 3\sqrt[3]{x^2} = 0$

$2x = 3\sqrt[3]{x^2}$

$(2x)^3 = (3\sqrt[3]{x^2})^3$

$8x^3 = 27x^2$

$x = \frac{27}{8}$

$f(8) =$
$2\cdot 8 - 3\sqrt[3]{8}^2 =$
$16 - 3\cdot 2^2 = 4$