

1. Consider the function $f(x) = x^2 - 4x + 7$.

- (a) Find all critical points of $f(x)$.

$$\begin{aligned} f'(x) &= 2x - 4 = 0 \\ x - 2 &= 0 \\ x &= 2 \end{aligned}$$

$f'(x)$ is defined for all x . $f'(x) = 0$ only for $x = 2$. Therefore
[$x = 2$ is critical point]

- (b) Find the absolute extrema of $f(x)$ on the closed interval $[0, 3]$.

$$\begin{aligned} f(0) &= 0^2 - 4 \cdot 0 + 7 = 7 \quad \leftarrow \text{largest} \\ f(2) &= 2^2 - 4 \cdot 2 + 7 = 3 \quad \leftarrow \text{smallest} \\ f(3) &= 3^2 - 4 \cdot 3 + 7 = 4 \end{aligned}$$

Absolute max: $f(0) = 7$
Absolute min: $f(2) = 3$

1. Consider the function $f(x) = x^3 - 3x$.

- (a) Find all critical points of $f(x)$.

$$\begin{aligned} f'(x) &= 3x^2 - 3 = 0 \\ x^2 - 1 &= 0 \\ (x-1)(x+1) &= 0 \end{aligned}$$

$f'(x) = 0$ for $x = -1$ and $x = 1$
[Critical points are 1 and -1]

- (b) Find the absolute extrema of $f(x)$ on the closed interval $[0, 2]$.

Only the critical point $x = 1$ is in the interval.

$$\begin{aligned} f(0) &= 0^3 - 3 \cdot 0 = 0 \\ f(1) &= 1^3 - 3 \cdot 1 = -2 \quad \leftarrow \text{smallest} \\ f(2) &= 2^3 - 3 \cdot 2 = 2 \quad \leftarrow \text{largest} \end{aligned}$$

Absolute max: $f(2) = 2$
Absolute min: $f(1) = -2$