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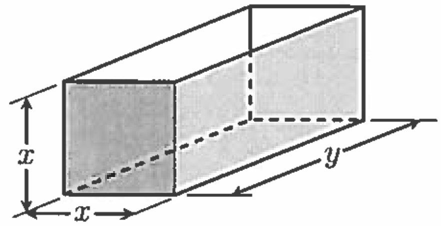
QUIZ 10

MATH 200, SECTION 1
April 16, 2021

1. (12 points) A metal box with two square ends and an open top is to contain a volume of 36 cubic inches. What dimensions x and y will minimize the total area of the metal surface?



We need to minimize surface area

$$\text{Surface area} = \underbrace{2x^2}_{\text{front \& back}} + \underbrace{2xy}_{\text{two sides}} + \underbrace{xy}_{\text{bottom}}$$

$$= 2x^2 + 3xy$$

Constraint Volume $= 36 = x \cdot x \cdot y$, so $\boxed{y = \frac{36}{x^2}}$

$$\text{Surface area} = 2x^2 + 3x\frac{36}{x^2}$$

So we seek the x that maximizes $S(x) = 2x^2 + \frac{108}{x}$ on the interval $(0, \infty)$

$$S'(x) = 4x - \frac{108}{x^2} = 0$$

$$4x = \frac{108}{x^2}$$

$$4x^3 = 108$$

$$x^3 = 27$$

$$x = \sqrt[3]{27} = 3 \leftarrow \text{critical point}$$

0   3

$- - - - \mid + + +$ $S'(x) = 4x - \frac{108}{x^2}$

Answer: For minimum surface area, use $x = 3$ and $y = \frac{36}{3^2} = \frac{36}{9} = 4$, so $\boxed{3 \times 3 \times 4}$

2. (8 points) Find $\lim_{x \to 0} \frac{e^{3x} - 1}{x^2 + 3x}$. $= \lim_{x \to 0} \frac{e^{3x} \cdot 3}{2x + 3} = \frac{e^{3 \cdot 0} \cdot 3}{2 \cdot 0 + 3} = \frac{3}{3} = \boxed{1}$

form $\frac{0}{0}$