Virginia Commonwealth University
Department of Mathematics

Math 200, Final Exam Review Problems

Your exam may contain problems that do not resemble these review problems.

(1) Evaluate the exact value of the following limits using any method.

(a) $\lim_{x\to 1} \dfrac{x^2+2x-3}{x^2-4x+3}$, $\lim_{x\to 3\pi} \dfrac{x+\sin(2x)}{4+\cos x}$

(b) $\lim_{x\to 3^+} \dfrac{x+2}{x-3}$, $\lim_{x\to 4^-} \dfrac{x-5}{x^2-16}$

(c) $\lim_{x\to 0} \dfrac{\sqrt{x+4}-2}{x}$, $\lim_{x\to 0} \dfrac{\sin(5x)}{\tan(7x)}$

(d) $\lim_{x\to\infty} \dfrac{x^3+4x^2+1}{45x^2+5x+10}$, $\lim_{x\to-\infty} \dfrac{2x^5+7x^2+3x+1}{14x^4+5x^3+5x+1}$

(e) $\lim_{x\to 1} \dfrac{\sin(\pi x)}{x^5-1}$, $\lim_{x\to 0} \dfrac{e^x-x-1}{3x^2}$, $\lim_{x\to\pi/2} \dfrac{1-\sin(x)+\cos(x)}{\sin(x)+\cos(x)-1}$

(f) $\lim_{x\to 0} \dfrac{e^x-1}{x+\sin x}$, $\lim_{x\to 0} \dfrac{x-\sin x}{x\tan x}$

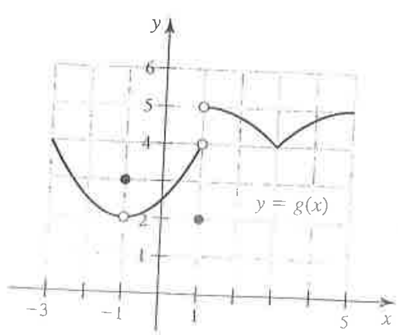
(g) $\lim_{x\to\infty} \dfrac{2e^x+3}{2-5e^x}$, $\lim_{x\to-\infty} \dfrac{2e^x+3}{2-5e^x}$, $\lim_{x\to 0+} x\left(5+\dfrac{3}{x}\right)$

(2) Find all values of x where $f(x) = \dfrac{\sin x}{x^3-2x}$ is continuous. Express your answer as interval(s).

(3) Finding Limits from a graph: Use the graph of g below to find the following limits. If the exist. If a limit does not exist, explain why.

(a) $\lim_{x\to -1} g(x)$, $\lim_{x\to 1} g(x)$, and $\lim_{x\to 3} g(x)$.

(b) Find values of x where g fails to be (a) continuous and (b) diferentaible in the interval $(-3, 5)$.



(4) Find numbers A and B that make $f(x)$ continuous for all values of x.

$$f(x) = \begin{cases} 2+x^2 & \text{if } x<1, \\ Ax+B & \text{if } 1 \le x \le 2 \\ 3x+5 & \text{if } x>2 \end{cases}$$

(5) Find the derivative of the following functions.

(a) $y = x^3 + 2x + \frac{1}{x}$, $\quad y = x^3 \ln x$, $\quad y = x \sin x \cos x$

(b) $f(x) = \frac{3}{x^{3/5}} + \sqrt[5]{x} + x^8 + 15$, $\quad g(x) = \ln(x^5) + \sin(3x^3) + e^{5x}$

(c) $f(\theta) = \sqrt{\theta}\tan(\theta)$ $\quad f(t) = (t^4 \ln t)^3$, $\quad f(x) = \sqrt[5]{x^2+1}$

(d) $f(x) = \frac{x \ln x}{1+\ln x}$, $\quad f(z) = \frac{z}{\sqrt[3]{z^2+1}}$, $\quad h(x) = \frac{1-\sin x}{1+\cos x}$

(e) $h(x) = x^2 \sin^{-1}(\sqrt{x})$, $\quad g(t) = \tan^{-1}(3t^2)$, $\quad p(x) = \cos^{-1}(2x)$

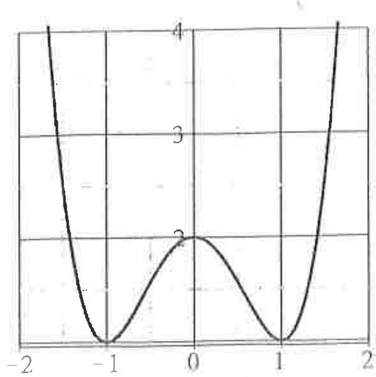
(f) $y = x^{\tan(x)}$, $\quad h(x) = \int_0^x \frac{1}{\sqrt{t^4+1}} dt$, $\quad g(t) = \int_3^{\cos(t)} e^{x^2} dx$

(6) Find an equation of the tangent line to the curve $y = \frac{2x}{x^2+1}$ at the point $(1,1)$.

(7) Find an equation of the tangent to the graph of $y = \cos(x)e^{-5x}$ at $x = 0$.

(8) Find an equation of the tangent line $x^2y^3 + xy = 78$ at the point $(3,2)$ on the curve.

(9) Given the graph of a function $y = f(x)$, sketch approximate graph of its derivative, $y = f'(x)$.



(a) graph of $y=f(x)$

(10) Supopose a stone is thrown vertically upward with an initial velocity of $64ft^2$ from a bridge 96 ft above a river. By Newton's law of motion, the position of the stone(measures as a height above the river) after t seconds is

$$s(t) = -16t^2 + 64t + 96$$

where $s = 0$ is the level of the river.

(a) Find the velocity and acceleration function
(b) What is the heghest point above the river reached by the stone?
(c) With what velocity willl the stone strike the river?

(11) If a tank holds 5000 gallons of water, which drains from the bottom of the tank in 40 minutes, then Torricelli's Law gives the volume of the water remaining in the tank after t minutes as

$$V(t) = 5000\left(1 - \frac{1}{40}t\right)^2, \qquad 0 \leq t \leq 40.$$

Find the rate at which water is draining from the tank after (a) 5 minutes (b) after 10 minutes.

(12) At what rate is the diagonal of a cube increasing if its edges are increasing at the rate of 2 cm/sec?

(13) A cild flies a kite at a height of 80 ft, the wind carring the kite horizontally away from the child at a rate of 34 ft/sec. How fasr must the child let out the string when the kite is 170 ft away from the child?

(14) A point moves along the curve $2x^2 - y^2 = 2$. When the point is at $(3, -4)$, its x-coordinate is increasing at the rate of 2 units per second. How fast is its y-coordinate changing at this time?

(15) Find the absolute maximum and absolute minimum values of $r(\theta) = \sin(\theta) + \cos(\theta)$ on the interval $[0, 2\pi]$.

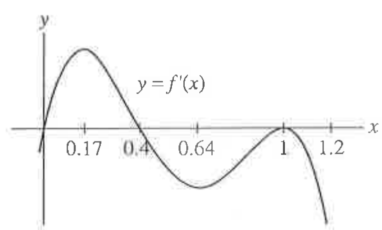
(16) Let $f(x) = x^4 - 2x^2 + 4$.

(a) Find the horizontal and vertical asymptotes of the graph of $f(x)$.
(b) Determine intervals on which f is increasing and intervals on which f is decreasing. Also find local maximum and local minimum values of f.
(c) Determine intervals on which f is concave up and intervals on which f is concave down. Also find points of inflection.
(d) Sketch the graph of f.

(17) Sketch the graph of a function that satisfies all of the following conditions

$f'(0) = f'(2) = f'(4) = 0$; $f'(x) > 0$ if $x < 0$ or $2 < x < 4$; $f'(x) < 0$ if $0 < x < 2$ or $x > 4$; $f''(x) > 0$ if $1 < x < 3$; $f''(x) < 0$ if $x < 1$ or $x > 3$.

(18) The graph of the derivative $f'(x)$ on the interval $[0, 1.2]$ is shown below. Answer the following questions based on this graph.
   (a) Find the critical points of f. Which critical points correspond to local maximum? Local minimum?
   (b) Where is f increasing? f decreasing?
   (c) Where is f concave up? f concave down?
   (d) Find the x coordinates of inflection point(s) of f.

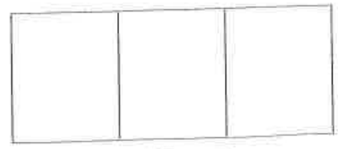


(19) A box has a square base of side x and height y. Find the dimensions x, y for which the volume is $12 ft^3$ and the surface area of the box is as small as possible.

(20) A farmer has 2400 ft of fencing and wants to fence off a rectangular field that borders a straight river. He needs no fence along the river. What are the dimensions of the field that has the largest area?

(21) If 1200 cm^2 of material is available to make a box with a square base and an open top, find the largest possible volume of the box.

(22) Consider a rectangular industrial warehouse consisting of three separate spaces of equal size as shown in the figure below. Assume that the wall material costs $\$200/ft$ and the company allocates $\$2,400,000$ for the project. Find the dimensions that maximizes the area of the warehouse. What is the maximum area?



(23) Find point(s) on the parabola $\frac{x^2}{4} + y^2 = 1$ closest to the point $(2, 0)$.

(24) Given an antiderivative of $y = \sin(x^2)$ in terms of indefinite integral.

(25) If the population of a bacteria is growing at an instantaneous rate of 2^t million bacteria per hour, what does the definite integral $\int_0^1 2^t\,dt$ represent? If $F(t)$ represent the number of bacteria at time t, what does $F(0) + \int_0^1 2^t\,dt$ represent?

(26) Find the following integral.

(a) $\int \left(5x^3 + \sec^2(x) + 6\cos(x) + \frac{2}{x} + \frac{1}{x^4} + 7e^{3x} - \pi^2\right) dx$

(b) $\int \frac{1}{x(\ln x)^3} dx$

(c) $\int \cos^4(x)\sin(x)dx$

(d) $\int \frac{\tan^{-1}(x)}{1+x^2}\,dx$

(e) $\int x^3\sqrt{x^2+3}dx$

(27) Evaluate the following definite integrals

(a) $\int_0^{\pi/2} (\cos(2x)+3)dx$

(b) $\int_4^9 \frac{x-\sqrt{x}}{x^3}dx$

(c) $\int_0^1 (x^3 + \sqrt[3]{x})dx$

(d) $\int_0^{2\pi} \sin^2(\theta)\,d\theta$

(e) $\int_0^2 x^3\sqrt{1+x^4}dx$

(f) $\int_0^{\pi/2} \frac{2\sin(2t)}{4-\cos(2t)}dt$

(28) The acceleration of an obeject moving along a line is given as $a(t) = 3\sin(2t)$ at any time t. Find the velocity function and the position function of the object at any time t if it is known that $v(0) = 1$ and $s(0) = 0$.

(29) (a) Find $y = y(x)$ if $\frac{d^2y}{dx} = 8x$, $\frac{dy}{dx}(0) = 2$, and $y(0) = 0$

(30) Find the area bounded between the graph of $y = \sin(x)$ and the x-axis from $x = 0$ to $x = 2\pi$.

(31) Find the area of the region bounded by the curves $y = x^2 - 4$ and $y = x + 2$.

(32) Find the area of the region bounded by the graphs of $y = x^2 - 1$ and $y = -x^2 + 3$.

(33) Find the area of the region bounded by the curves $x = y^2$ and $y = x - 2$.