Virginia Commonwealth University
Department of Mathematics

Math 200, Solutiok Key, Final Exam Review Problems

Your exam may contain problems that do not resemble these review problems.

(1) (a) -2;π
(b) $\infty;\infty$
(c) 1/4 ; 5/7
(d) $\infty,\ -\infty$;
(e) $-\pi/5$; 1/6; 1
(f) 1/2;0
(g) -2/5; 3/2; 3

(2) f is continuous on $(-\infty,-\sqrt{2})\cup(-\sqrt{2},0)\cup(0,\sqrt{2})\cup(\sqrt{2},\infty)$.

(3) (a) 2; DNE;4
(b) (i) $g(x)$ is not continuous at $x=-1$ since $2=\lim_{x\to-1} g(x)\neq g(-1)=3$.
$g(x)$ is not continuous at $x=1$ since $\lim_{x\to1} g(x)$ DNE.

(ii) f is not differentiable at -1,1, and 3

(4) $A=8$ and $B=-5$.

(5) (a) $y'=3x^2+2-\dfrac{1}{x^2}$; $y'=3x^2\ln(x)+x^2$;$y'=\sin(x)\cos(x)+x\cos^2(x)-x\sin^2(x)$
(b) $f'(x)=-\dfrac{9}{5}x^{-8/5}+\dfrac{1}{5}x^{-4/5}+8x^7$; $g'(x)=\dfrac{5}{x}+9x^2\cos(3x^3)+5e^{5x}$
(c) $g'(\theta)=\dfrac{1}{2}\theta^{-1/2}\tan(\theta)+\sqrt{\theta}\sec^2(\theta)$; $f'(t)=3(t^4\ln(t))^2(4t^3\ln(t)+t^3)$;

$f'(x)=\dfrac{1}{5}(x^2+1)^{-4/5}2x$

(d) $f'(x)=\dfrac{(\ln(x)+1)(1+\ln(x))-\ln(x)}{(1+\ln(x))^2}$

$f'(z)=\dfrac{\sqrt[3]{z^2+1}-\frac{2}{3}z^2(z^2+1)^{-2/3}}{(z^2+1)^{2/3}}$

$h'(x)=\dfrac{-\cos(x)(1+\cos(x))-(1-\sin(x))(-\sin(x))}{(1+\cos(x))^2}$

(e) $h'(x)=2x\arcsin(\sqrt{x})+x^2\dfrac{1}{\sqrt{1-x}}\dfrac{1}{2\sqrt{x}}$;

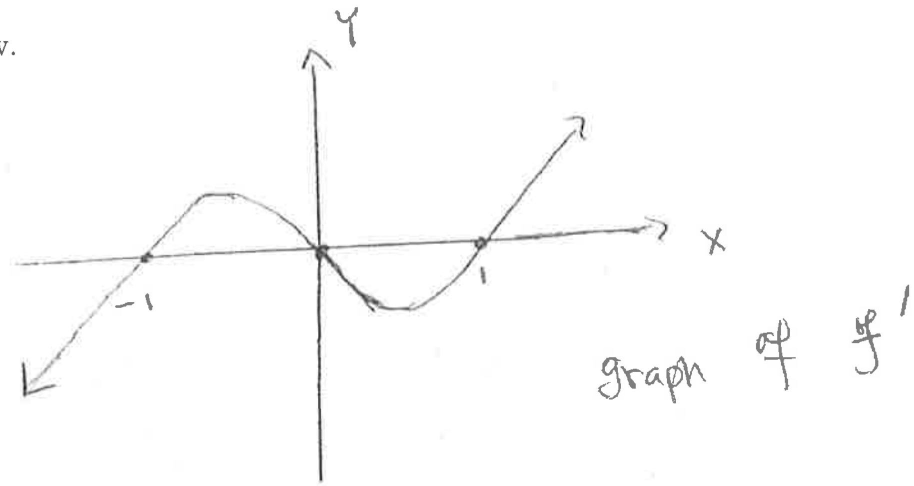
$g'(t)=\dfrac{6t}{1+9t^4}$

$p'(z)=\dfrac{-2}{\sqrt{1-4x^2}}$

(f) $f'(x)=x^{\tan(x)}\left(\sec^2(x)\ln(x)+\dfrac{\tan(x)}{x}\right)$

$$h'(x) = \frac{1}{\sqrt{x^4+1}}$$
$$g'(x) = e^{\cos^2(t)}(-\sin(t))$$

(6) $y = 1$
(7) $y = -5x + 1$
(8) $y - 2 = \frac{-50}{111}(x - 3)$
(9) See the figure below.



(10) (a) $v(t) = -32t + 64, \ t \geq 0; a(t) = -32 \ t \geq 0$
  (b) 160 ft
  (c) $-32\sqrt{10} ft/sec$
(11) (a) -218.75 ft^3/sec (b) -187.5 ft^3/sec
(12) $2\sqrt{3} ft/sec$
(13) 30 ft/sec
(14) -3 units /sec
(15) Absolute maximum value = $\sqrt{2}$ and absolute minimum value=-$\sqrt{2}$

(16) (ii)
  (a) NO HA and NO VA
  (b) f is increasing on $(-1, 0) \cup (1, \infty)$, and f is decreasing on $(-\infty, -1) \cup [0, 1)$

  $f(0) = 4$ is a local maximum value and $f(1) = f(-1) = 3$ is a local minimum value of f
  (c) f is concave up on $(-\infty, -\frac{1}{\sqrt{3}}) \cup (\frac{1}{\sqrt{5}}, \infty)$ and concave down on $(-\frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}})$. point of inflections are $(-\frac{1}{\sqrt{3}}, 31/9)$ and $(\frac{1}{\sqrt{3}}, 31/9)$
(17) Exercise
(18) (a) Critical points are 0.4 and 1. $x = 0.4$ corresponds to l. maximum value and $x = 1$ does not correspond to local extrema.
  (b) f is increasing on $[0, 0.4]$ and f is decreasing on $[0.4, 1.2]$.
  (c) f is concave up on $(0.0.17) \cup (0.64, 1)$ and concave down on $(0.17, 0.64) \cup (1.1.2)$.

f has inflection points at $x = 0.17, x = 0.64$ and $x = 1$.

(19) $x = y = \sqrt[3]{12}$

(20) A height of 600 ft and a width of 1200 ft.

(21) The maximum volume is 4000 cubic feet when the base is 20 ft and the height is 10 ft.

(22) A height of 1500ft and a total width of 3000ft.

(23) The point is $(2, 0)$.

(24) $F(x) = \int_0^x \sin(t^2)dt$.

(25) $\int_0^1 2^t\,dt$= Total change in population during the first hour. $F(0) + \int_0^1 2^t dt =$ Total population at the end of the first hour.

(26) (a) $\frac{5}{4}x^4 + \tan(x) + 6\sin(x) + 2\ln(|x|) - \frac{1}{3x^3} + \frac{7}{3}e^{3x} - \pi^2 x + C$

(b) $-\frac{1}{2(\ln|x|)^2} + C$

(c) $-\frac{1}{5}\cos^5(x) + C$

(d) $1/2arctan^2(x)| + C$

(e) $\frac{1}{5}(x^2+3)^{5/2} - (x^2+3)^{3/2} + C$

(27) (a) $3\pi/2$

(b) $13/162$

(c) 1

(d) π

(e) $\frac{17}{6}\sqrt{17} - \frac{1}{6}$

(f) $\ln 5 - \ln 3$

(28) $v(t) = -\frac{3}{2}\cos(2t) + \frac{5}{2},\ t \geq 0$; $s(t) = -\frac{3}{4}\sin(2t) + \frac{5}{2}t,\ t \geq 0$

(29) $y(x) = \frac{4}{3}x^3 + 2x$

(30) 4 square units

(31) $A = \int_{-2}^{3} (x + 2 - (x^2 - 4))\,dx = \frac{125}{6}$ square units.

(32) $A = \int_{-\sqrt{2}}^{\sqrt{2}} (-x^2 + 3 - (x^2 - 1))\,dx = \frac{16}{3}\sqrt{2}$ square units.

(33) $A = \int_{-1}^{2} (y + 2 - y^2)\,dy = \frac{9}{2}$ square units.